============== Title

Spherically symmetric line element with perturbation a/r

============== Coordinates x[0], x[1], x[2], x[3] =

$$t$$

$$r$$

$$\vartheta$$

$$\varphi$$

============== Metric g =

$$\begin{pmatrix} \frac{\frac{2\,G\,M}{c^2}+\frac{a}{r}}{r}+1 & 0 & 0 & 0 \\ 0 & \frac{1}{\frac{\frac{2\,G\,M}{c^2}+\frac{a}{r}}{r}+1} & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2\,\sin^2\vartheta \end{pmatrix}$$

============== Contravariant Metric gContr =

$$\begin{pmatrix} \frac{c^2\,r^2}{2\,r\,G\,M+c^2\,r^2+a\,c^2} & 0 & 0 & 0 \\ 0 & \frac{2\,r\,G\,M+c^2\,r^2+a\,c^2}{c^2\,r^2} & 0 & 0 \\ 0 & 0 & \frac{1}{r^2} & 0 \\ 0 & 0 & 0 & \frac{1}{r^2\,\sin^2\vartheta} \end{pmatrix}$$

============== Christoffel Connection Gamma

$\Gamma[0,0,1] =$

$$-\frac{r\,G\,M+a\,c^2}{r\,\left(2\,r\,G\,M+c^2\,r^2+a\,c^2\right)}$$

$\Gamma[0,1,0] =$

$$-\frac{r\,G\,M+a\,c^2}{r\,\left(2\,r\,G\,M+c^2\,r^2+a\,c^2\right)}$$

$\Gamma[1,0,0] =$

$$\frac{\left(r\,G\,M+a\,c^2\right)\,\left(2\,r\,G\,M+c^2\,r^2+a\,c^2\right)}{c^4\,r^5}$$

$\Gamma[1,1,1] =$

$$\frac{r\,G\,M+a\,c^2}{r\,\left(2\,r\,G\,M+c^2\,r^2+a\,c^2\right)}$$

$\Gamma[1,2,2] =$

$$-\frac{2\,r\,G\,M+c^2\,r^2+a\,c^2}{c^2\,r}$$

$\Gamma[1,3,3] =$

$$-\frac{\sin^2\vartheta\,\left(2\,r\,G\,M+c^2\,r^2+a\,c^2\right)}{c^2\,r}$$

$\Gamma[2,1,2] =$

$$\frac{1}{r}$$

Γ[2, 2, 1] =

$$\frac{1}{r}$$

Γ[2, 3, 3] =

$$-\cos\vartheta\,\sin\vartheta$$

Γ[3, 1, 3] =

$$\frac{1}{r}$$

Γ[3, 2, 3] =

$$\frac{\cos\vartheta}{\sin\vartheta}$$

Γ[3, 3, 1] =

$$\frac{1}{r}$$

Γ[3, 3, 2] =

$$\frac{\cos\vartheta}{\sin\vartheta}$$

============== Metric compatibility
============== o.k.

============== Riemann Tensor
R[0,1,0,1] =

$$-\frac{2\,r\,G\,M+3\,a\,c^2}{r^2\,\left(2\,r\,G\,M+c^2\,r^2+a\,c^2\right)}$$

R[0,1,1,0] =

$$\frac{2\,r\,G\,M+3\,a\,c^2}{r^2\,\left(2\,r\,G\,M+c^2\,r^2+a\,c^2\right)}$$

R[0,2,0,2] =

$$\frac{r\,G\,M+a\,c^2}{c^2\,r^2}$$

R[0,2,2,0] =

$$-\frac{r\,G\,M+a\,c^2}{c^2\,r^2}$$

R[0,3,0,3] =

$$\frac{\sin^2\vartheta\,\left(r\,G\,M+a\,c^2\right)}{c^2\,r^2}$$

R[0,3,3,0] =

$$-\frac{\sin^2\vartheta\,\left(r\,G\,M+a\,c^2\right)}{c^2\,r^2}$$

R[1,0,0,1] =

$$\frac{\left(2\,r\,G\,M+3\,a\,c^2\right)\,\left(2\,r\,G\,M+c^2\,r^2+a\,c^2\right)}{c^4\,r^6}$$

R[1,0,1,0] =

$$-\frac{\left(2\,r\,G\,M+3\,a\,c^2\right)\,\left(2\,r\,G\,M+c^2\,r^2+a\,c^2\right)}{c^4\,r^6}$$

R[1,2,1,2] =

$$\frac{r\,G\,M + a\,c^2}{c^2\,r^2}$$

R[1,2,2,1] =

$$-\frac{r\,G\,M + a\,c^2}{c^2\,r^2}$$

R[1,3,1,3] =

$$\frac{\sin^2\vartheta\,\left(r\,G\,M + a\,c^2\right)}{c^2\,r^2}$$

R[1,3,3,1] =

$$-\frac{\sin^2\vartheta\,\left(r\,G\,M + a\,c^2\right)}{c^2\,r^2}$$

R[2,0,0,2] =

$$-\frac{\left(r\,G\,M + a\,c^2\right)\,\left(2\,r\,G\,M + c^2\,r^2 + a\,c^2\right)}{c^4\,r^6}$$

R[2,0,2,0] =

$$\frac{\left(r\,G\,M + a\,c^2\right)\,\left(2\,r\,G\,M + c^2\,r^2 + a\,c^2\right)}{c^4\,r^6}$$

R[2,1,1,2] =

$$-\frac{r\,G\,M + a\,c^2}{r^2\,\left(2\,r\,G\,M + c^2\,r^2 + a\,c^2\right)}$$

R[2,1,2,1] =

$$\frac{r\,G\,M + a\,c^2}{r^2\,\left(2\,r\,G\,M + c^2\,r^2 + a\,c^2\right)}$$

R[2,3,2,3] =

$$-\frac{\sin^2\vartheta\,\left(2\,r\,G\,M + a\,c^2\right)}{c^2\,r^2}$$

R[2,3,3,2] =

$$\frac{\sin^2\vartheta\,\left(2\,r\,G\,M + a\,c^2\right)}{c^2\,r^2}$$

R[3,0,0,3] =

$$-\frac{\left(r\,G\,M + a\,c^2\right)\,\left(2\,r\,G\,M + c^2\,r^2 + a\,c^2\right)}{c^4\,r^6}$$

R[3,0,3,0] =

$$\frac{\left(r\,G\,M + a\,c^2\right)\,\left(2\,r\,G\,M + c^2\,r^2 + a\,c^2\right)}{c^4\,r^6}$$

R[3,1,1,3] =

$$-\frac{r\,G\,M + a\,c^2}{r^2\,\left(2\,r\,G\,M + c^2\,r^2 + a\,c^2\right)}$$

R[3,1,3,1] =

$$\frac{r\,G\,M + a\,c^2}{r^2\,\left(2\,r\,G\,M + c^2\,r^2 + a\,c^2\right)}$$

R[3,2,2,3] =

$$\frac{2\,r\,G\,M + a\,c^2}{c^2\,r^2}$$

R[3,2,3,2] =

$$-\frac{2\,r\,G\,M + a\,c^2}{c^2\,r^2}$$

============== Ricci Tensor

Ric[0,0] =

$$-\frac{a\,\left(2\,r\,G\,M + c^2\,r^2 + a\,c^2\right)}{c^2\,r^6}$$

Ric[1,1] =

$$-\frac{a\,c^2}{r^2\,\left(2\,r\,G\,M + c^2\,r^2 + a\,c^2\right)}$$

Ric[2,2] =

$$\frac{a}{r^2}$$

Ric[3,3] =

$$\frac{a\,\sin^2\vartheta}{r^2}$$

============== Ricci Scalar

R_{sc} =

$$0$$

============== Bianchi identity (Ricci cyclic equation $R \wedge q = 0$)

============== o.k.

============== Einstein tensor

============== not zero:

G[0,0] =

$$-\frac{a\,\left(2\,r\,G\,M + c^2\,r^2 + a\,c^2\right)}{c^2\,r^6}$$

G[1,1] =

$$-\frac{a\,c^2}{r^2\,\left(2\,r\,G\,M + c^2\,r^2 + a\,c^2\right)}$$

G[2,2] =

$$\frac{a}{r^2}$$

G[3,3] =

$$\frac{a\,\sin^2\vartheta}{r^2}$$

============== Bianchi identity of Hodge dual

============== (see charge and current densities)

============== Scalar charge density

$\rho =$

$$-\frac{a\,c^2}{r^2\,\left(2\,r\,G\,M+c^2\,r^2+a\,c^2\right)}$$

============= Current density class 1

J[1] =

$$\frac{4\,r^2\,G^2\,M^2+\left(2\,c^2\,r^3+8\,a\,c^2\,r\right)\,G\,M+3\,a\,c^4\,r^2+3\,a^2\,c^4}{c^4\,r^6}-\frac{2\,\left(2\,r^2\,G^2\,M^2+\left(c^2\,r^3+3\,a\,c^2\,r\right)\,G\,M+a\,c^4\,r^2+\right.}{c^4\,r^6}$$

J[2] =

$$\frac{2\,r\,G\,M+a\,c^2}{c^2\,r^6}-\frac{2\,\left(r\,G\,M+a\,c^2\right)}{c^2\,r^6}$$

J[3] =

$$\frac{2\,r\,G\,M+a\,c^2}{c^2\,r^6\,\sin^2\vartheta}-\frac{2\,\left(r\,G\,M+a\,c^2\right)}{c^2\,r^6\,\sin^2\vartheta}$$

============= Current density class 2

J[1] =

$$0$$

J[2] =

$$0$$

J[3] =

$$0$$

============= Current density class 3

J[1] =

$$0$$

J[2] =

$$0$$

J[3] =

$$0$$