============== Title

Spherically symmetric line element with general mu(r)

============== Coordinates x[0], x[1], x[2], x[3] =

$$t$$

$$r$$

$$\vartheta$$

$$\varphi$$

============== Metric g =

$$\begin{pmatrix} \frac{\mu}{r}+1 & 0 & 0 & 0 \\ 0 & \frac{1}{\frac{\mu}{r}+1} & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2\,\sin^2\vartheta \end{pmatrix}$$

============== Contravariant Metric gContr =

$$\begin{pmatrix} \frac{r}{r+\mu} & 0 & 0 & 0 \\ 0 & \frac{r+\mu}{r} & 0 & 0 \\ 0 & 0 & \frac{1}{r^2} & 0 \\ 0 & 0 & 0 & \frac{1}{r^2\,\sin^2\vartheta} \end{pmatrix}$$

============== Christoffel Connection Gamma

$\Gamma[0,0,1] =$

$$\frac{\frac{d}{d\,r}\,\mu\,r-\mu}{2\,r\,\left(r+\mu\right)}$$

$\Gamma[0,1,0] =$

$$\frac{\frac{d}{d\,r}\,\mu\,r-\mu}{2\,r\,\left(r+\mu\right)}$$

$\Gamma[1,0,0] =$

$$-\frac{\left(r+\mu\right)\,\left(\frac{d}{d\,r}\,\mu\,r-\mu\right)}{2\,r^3}$$

$\Gamma[1,1,1] =$

$$-\frac{\frac{d}{d\,r}\,\mu\,r-\mu}{2\,r\,\left(r+\mu\right)}$$

$\Gamma[1,2,2] =$

$$-\left(r+\mu\right)$$

$\Gamma[1,3,3] =$

$$-\left(r+\mu\right)\,\sin^2\vartheta$$

$\Gamma[2,1,2] =$

$$\frac{1}{r}$$

$\Gamma[2,2,1] =$

$$\frac{1}{r}$$

$\Gamma[2,3,3] =$

$$-\cos\vartheta\,\sin\vartheta$$

$\Gamma[3,1,3] =$

$$\frac{1}{r}$$

$\Gamma[3,2,3] =$

$$\frac{\cos\vartheta}{\sin\vartheta}$$

$\Gamma[3,3,1] =$

$$\frac{1}{r}$$

$\Gamma[3,3,2] =$

$$\frac{\cos\vartheta}{\sin\vartheta}$$

============== Metric compatibility
============== o.k.

============== Riemann Tensor
R[0,1,0,1] =

$$-\frac{\frac{d^2}{d\,r^2}\,\mu\,r^2 - 2\,\left(\frac{d}{d\,r}\,\mu\right)\,r + 2\,\mu}{2\,r^2\,(r+\mu)}$$

R[0,1,1,0] =

$$\frac{\frac{d^2}{d\,r^2}\,\mu\,r^2 - 2\,\left(\frac{d}{d\,r}\,\mu\right)\,r + 2\,\mu}{2\,r^2\,(r+\mu)}$$

R[0,2,0,2] =

$$-\frac{\frac{d}{d\,r}\,\mu\,r - \mu}{2\,r}$$

R[0,2,2,0] =

$$\frac{\frac{d}{d\,r}\,\mu\,r - \mu}{2\,r}$$

R[0,3,0,3] =

$$-\frac{\left(\frac{d}{d\,r}\,\mu\,r - \mu\right)\,\sin^2\vartheta}{2\,r}$$

R[0,3,3,0] =

$$\frac{\left(\frac{d}{d\,r}\,\mu\,r - \mu\right)\,\sin^2\vartheta}{2\,r}$$

R[1,0,0,1] =

$$\frac{(r+\mu)\,\left(\frac{d^2}{d\,r^2}\,\mu\,r^2 - 2\,\left(\frac{d}{d\,r}\,\mu\right)\,r + 2\,\mu\right)}{2\,r^4}$$

R[1,0,1,0] =

$$-\frac{(r+\mu)\,\left(\frac{d^2}{d\,r^2}\,\mu\,r^2 - 2\,\left(\frac{d}{d\,r}\,\mu\right)\,r + 2\,\mu\right)}{2\,r^4}$$

R[1,2,1,2] =

$$-\frac{\frac{d}{d\,r}\,\mu\,r-\mu}{2\,r}$$

R[1,2,2,1] =

$$\frac{\frac{d}{d\,r}\,\mu\,r-\mu}{2\,r}$$

R[1,3,1,3] =

$$-\frac{\left(\frac{d}{d\,r}\,\mu\,r-\mu\right)\,\sin^2\vartheta}{2\,r}$$

R[1,3,3,1] =

$$\frac{\left(\frac{d}{d\,r}\,\mu\,r-\mu\right)\,\sin^2\vartheta}{2\,r}$$

R[2,0,0,2] =

$$\frac{(r+\mu)\,\left(\frac{d}{d\,r}\,\mu\,r-\mu\right)}{2\,r^4}$$

R[2,0,2,0] =

$$-\frac{(r+\mu)\,\left(\frac{d}{d\,r}\,\mu\,r-\mu\right)}{2\,r^4}$$

R[2,1,1,2] =

$$\frac{\frac{d}{d\,r}\,\mu\,r-\mu}{2\,r^2\,(r+\mu)}$$

R[2,1,2,1] =

$$-\frac{\frac{d}{d\,r}\,\mu\,r-\mu}{2\,r^2\,(r+\mu)}$$

R[2,3,2,3] =

$$-\frac{\mu\,\sin^2\vartheta}{r}$$

R[2,3,3,2] =

$$\frac{\mu\,\sin^2\vartheta}{r}$$

R[3,0,0,3] =

$$\frac{(r+\mu)\,\left(\frac{d}{d\,r}\,\mu\,r-\mu\right)}{2\,r^4}$$

R[3,0,3,0] =

$$-\frac{(r+\mu)\,\left(\frac{d}{d\,r}\,\mu\,r-\mu\right)}{2\,r^4}$$

R[3,1,1,3] =

$$\frac{\frac{d}{d\,r}\,\mu\,r-\mu}{2\,r^2\,(r+\mu)}$$

R[3,1,3,1] =

$$-\frac{\frac{d}{d\,r}\,\mu\,r-\mu}{2\,r^2\,(r+\mu)}$$

R[3,2,2,3] =

$$\frac{\mu}{r}$$

R[3,2,3,2] =

$$-\frac{\mu}{r}$$

============== Ricci Tensor

Ric[0,0] =

$$-\frac{\frac{d^2}{d\,r^2}\,\mu\,(r+\mu)}{2\,r^2}$$

Ric[1,1] =

$$-\frac{\frac{d^2}{d\,r^2}\,\mu}{2\,(r+\mu)}$$

Ric[2,2] =

$$-\frac{d}{d\,r}\,\mu$$

Ric[3,3] =

$$-\frac{d}{d\,r}\,\mu\,\sin^2\vartheta$$

============== Ricci Scalar

R_{sc} =

$$-\frac{\frac{d^2}{d\,r^2}\,\mu\,r+2\,\left(\frac{d}{d\,r}\,\mu\right)}{r^2}$$

============== Bianchi identity (Ricci cyclic equation $R \wedge q = 0$)

============== o.k.

============== Einstein tensor

============== not zero:

G[0,0] =

$$\frac{\left(\frac{\mu}{r}+1\right)\,\left(\frac{d^2}{d\,r^2}\,\mu\,r+2\,\left(\frac{d}{d\,r}\,\mu\right)\right)}{2\,r^2}-\frac{\frac{d^2}{d\,r^2}\,\mu\,(r+\mu)}{2\,r^2}$$

G[1,1] =

$$\frac{\frac{d^2}{d\,r^2}\,\mu\,r+2\,\left(\frac{d}{d\,r}\,\mu\right)}{2\,\left(\frac{\mu}{r}+1\right)\,r^2}-\frac{\frac{d^2}{d\,r^2}\,\mu}{2\,(r+\mu)}$$

G[2,2] =

$$\frac{\frac{d^2}{d\,r^2}\,\mu\,r+2\,\left(\frac{d}{d\,r}\,\mu\right)}{2}-\frac{d}{d\,r}\,\mu$$

G[3,3] =

$$\frac{\left(\frac{d^2}{d\,r^2}\,\mu\,r+2\,\left(\frac{d}{d\,r}\,\mu\right)\right)\,\sin^2\vartheta}{2}-\frac{d}{d\,r}\,\mu\,\sin^2\vartheta$$

============== Bianchi identity of Hodge dual

============== (see charge and current densities)

============== Scalar charge density

ρ =

$$-\frac{\frac{d^2}{d\,r^2}\,\mu}{2\,(r+\mu)}$$

============== Current density class 1

J[1] =

$$\frac{\frac{d^2}{d\,r^2}\,\mu\,r^3+\left(\mu\,\left(\frac{d^2}{d\,r^2}\,\mu\right)-2\,\left(\frac{d}{d\,r}\,\mu\right)\right)\,r^2+\left(2\,\mu-2\,\mu\,\left(\frac{d}{d\,r}\,\mu\right)\right)\,r+2\,\mu^2}{2\,r^4}+\frac{\frac{d}{d\,r}\,\mu\,r^2+\left(\mu\,\left(\frac{d}{d\,r}\,\mu\right)-\mu\right)\,r-\mu^2}{r^4}$$

J[2] =

$$\frac{\frac{d}{d\,r}\,\mu\,r-\mu}{r^5}+\frac{\mu}{r^5}$$

J[3] =

$$\frac{\frac{d}{d\,r}\,\mu\,r-\mu}{r^5\,\sin^2\vartheta}+\frac{\mu}{r^5\,\sin^2\vartheta}$$

============== Current density class 2

J[1] =

$$0$$

J[2] =

$$0$$

J[3] =

$$0$$

============== Current density class 3

J[1] =

$$0$$

J[2] =

$$0$$

J[3] =

$$0$$