Chapter 16

Chirality and spin vectors in ECE Theory

(Paper 70)
by
Myron W. Evans
Alpha Institute for Advanced Study (AIAS).
(emyrone@aol.com, www.aias.us, www.atomicprecision.com)

Abstract

The fundamental chirality and spin vectors of ECE theory are identified using the basic definition of the tetrad field as the rank two mixed index tensor that links two column vectors. The chirality vector defines the direction of spin (left or right handedness or chirality) and the spin vector indicates the existence of spin through a phase factor. The tetrad tensor is therefore made up of both handedness and spin, for example the components of the electromagnetic potential field are components of the tetrad tensor within a scalar factor $A^{(0)}$ where $cA^{(0)}$ is the primordial voltage of ECE theory. Similarly the fermion field is defined by a chirality two-spinor and a spin two-spinor. For the fermion, the tetrad field is a 2×2 mixed index tensor. The weak and strong fields can be developed similarly in terms of fundamental chirality and spin column vectors. Each field can be represented using inter-convertible representation spaces, for example the space part of the electromagnetic field can be represented by the $O(3)$ or $SU(n)$ groups, were $n = 2, .., n$. The $SU(2)$ representation of the electromagnetic field is the Majorana representation. This allows for field unification in any representation space.

Keywords: Einstein Cartan Evans (ECE) field theory, handedness, chirality, spin.

16.1 Introduction

In Einstein Cartan Evans (ECE) field theory [1]– [8] the fundamental field is the tetrad, which is a rank two mixed index tensor [9] that transforms as such under the general coordinate transformation, and is thus generally covariant. Therefore the fundamental fields of physics are tetrads of various kinds: the gravitational, electromagnetic, weak , strong and matter fields. The tetrad is defined as follows:

$$V^a = q^a_\mu V^\mu \tag{16.1}$$

where V^a and V^μ are column vectors which are also generally covariant. The tetrad field is therefore defined by the way in which V^a and V^μ are related geometrically, and the tetrad in turn defines the torsion tensor $T^a{}_{\mu\nu}$. In ECE theory the electromagnetic field for example is defined by the ansatz:

$$A^a_\mu = A^{(0)} q^a_\mu \tag{16.2}$$

$$F^a{}_{\mu\nu} = A^{(0)} T^a{}_{\mu\nu} \tag{16.3}$$

where $cA^{(0)}$ is the primordial voltage, c being the speed of light in vacuo and $A^{(0)}$ the potential magnitude of the electromagnetic field. The gravitational field is also defined by the tetrad, the symmetric metric being:

$$g_{\mu\nu} = q^a_\mu q^b_\nu \eta_{ab} \tag{16.4}$$

where η_{ab} is the metric in the tangent spacetime of Cartan geometry [1]– [9] at point P in the base manifold. In Section 16.2, V^a is defined as the chirality vector, and V^μ as the spin vector for the electromagnetic and fermion fields. The electromagnetic tetrad A^a_μ is therefore made up both of chirality (handedness) and spin - it can be left or right circularly polarized for example. Components of the tetrad tensor A^a_μ are denoted [1]– [9] $A^{(1)}_X$, and so on, and are components of the electromagnetic potential field. The electromagnetic field tensor is then defined by the first Cartan structure equation:

$$F^a{}_{\mu\nu} = (d \wedge A^a)_{\mu\nu} + (\omega^a{}_b \wedge A^b)_{\mu\nu} \tag{16.5}$$

where $\omega^a{}_b$ is the spin connection. For free rotation [1]– [8], the spin connection is dual to the tetrad:

$$\omega^a{}_{\mu b} = -\frac{\kappa}{2} \epsilon^a{}_{bc} q^c_\mu \tag{16.6}$$

and the spin connection can therefore be identified as being itself a potential field component. In this special case of pure rotation the spin connection becomes a generally covariant mixed index tensor (a tetrad tensor). In general however the spin connection is not a tensor [9]. Similarly the Christoffel connection of Riemann geometry is not a tensor in general because it does not transform covariantly under the general coordinate transformation. The tetrad in contrast always transforms covariantly because it is a rank two mixed-index tensor [9]. In general relativity any quantity with this property of general covariance may be a physical quantity (for example the Riemann tensor and the metric). In the standard model in contrast the electromagnetic potential field is a vector (i.e rank one tensor) A_μ and is developed with gauge theory in which it is not gauge invariant. A great deal of confusion results in the standard model for this reason, because it is held that a quantity that is not gauge invariant is not a physical quantity, a view that predates quantum theory and relativity and goes back to Heaviside. Faraday and Maxwell in contrast regarded A_μ as physical. At the same time in the standard model, A_μ is considered physical in the minimal prescription. The standard model is therefore self-inconsistent, in that A_μ is at once non-physical and physical, and is also incomplete, because it is special relativity, i.e. Lorentz covariant but not generally covariant as required by Einsteinian general relativity. ECE theory clears up this confusion by regarding A_μ as a generally covariant tetrad field, which is always a physical field. In ECE theory the tetrad is also the gravitational field, and in the latter, a is the index that defines the Minkowski or flat tangent spacetime of Cartan geometry [1]–[9] and μ is the index of a curving base manifold. As seen in Eq.(16.4), the symmetric metric of gravitational theory is made up of two tetrads multiplied together. The tetrad is therefore the fundamental gravitational field, and not the symmetric metric. The gravitational tetrad is therefore defined as the rank two tensor that links the flat spacetime column four-vector V^a with the curved spacetime column four-vector V^μ. These are four-vectors because there are four dimensions, time and three space dimensions. The gravitational tetrad therefore has 16 components. In the well known Einstein Hilbert (EH) theory there is no consideration given to torsion, only to curvature. For this reason EH is not a unified field theory as is well known. ECE is a unified field theory in the well defined sense that it is governed not by Riemann geometry without torsion, but by Cartan geometry with inclusion of both the Riemann or curvature form $R^a{}_b$ and the Cartan torsion form T^a. These are governed by the two well known Cartan structure equations:

$$T^a = D \wedge q^a := d \wedge q^a + \omega^a{}_b \wedge q^b \tag{16.7}$$

$$R^a{}_b = D \wedge \omega^a{}_b := d \wedge \omega^a{}_b + \omega^a{}_c \wedge \omega^c{}_b \tag{16.8}$$

and the two Bianchi identities of Cartan (i.e. differential) geometry:

$$D \wedge T^a := R^a{}_b \wedge q^b \tag{16.9}$$

$$D \wedge R^a{}_b := 0 \tag{16.10}$$

It is seen that the curvature and torsion are inter-related ineluctably by the basic geometry. The EH theory is the limit:

$$T^a = 0 \tag{16.11}$$

In Section 16.2 therefore the electromagnetic and fermion fields are developed as tetrad fields governed by Eqs.(16.7) to (16.10), and thus linked to the gravitational field by these equations of Cartan geometry, thus synthesizing a generally covariant unified field theory as required by the basic philosophy of objectivity (Bacon) and relativity (Einstein and others). In so doing, Section 16.2 defines the index a of the electromagnetic and fermion fields as that of chirality and the index μ of these fields as that of spin. Therefore the same overall method is used for the gravitational, electromagnetic and fermion fields, in that the tetrad definition links one column vector to another. For the fermion field, the $SU(2)$ representation space is used as is well known, and so the column vectors have two entries, i.e. are two-spinors. The spinor field is therefore a 2×2 tetrad with four components. The tetrad is therefore one two-component row vector superimposed on another. If each row vector is transposed to a column two-vector, the result is a column four-vector, the Dirac spinor [1]– [8] made up of two Pauli spinors. It is shown in Section 16.2 that the a index of the tetrad in this case represents handedness (right or left fermion) and the μ index represents spin. The Dirac spinor contains chirality (referred to in this case as helicity). The effect of any other field on the fermion field is then governed by the geometry of Eqs.(16.7) to (16.10) and by the minimal prescription. Finally Section 16.3 is a discussion of how these concepts can be extended to the weak and strong fields using the appropriate representation spaces, and how fields can be inter-related using ECE theory using any representation space such as $O(3)$ or $SU(n)$.

16.2 Definition of the chirality and spin vectors

We first review the development [1] of the Cartesian vector:

$$\mathbf{R} = X\mathbf{i} + Y\mathbf{j} + Z\mathbf{k} \tag{16.12}$$

in the $SU(2)$ basis, giving:

$$\begin{aligned} R = \boldsymbol{\sigma} \cdot \mathbf{R} = X\sigma_1 + Y\sigma_2 + Z\sigma_3 = \\ = \begin{bmatrix} Z & X - iY \\ X + iY & -Z \end{bmatrix} \end{aligned} \tag{16.13}$$

Thus:

$$R^2 = X^2 + Y^2 + Z^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{16.14}$$

The $SU(2)$ group is that of the unitary, unimodular matrices:

$$UU^+ = 1, \quad det\, U = 1 \tag{16.15}$$

which have the general form:

$$U = \begin{bmatrix} a & b \\ -b^* & a^* \end{bmatrix} \tag{16.16}$$

with

$$aa^* + bb^* = 1 \tag{16.17}$$

Define the two component spinor with complex valued elements:

$$\zeta = \begin{bmatrix} \zeta_1 \\ \zeta_2 \end{bmatrix} \tag{16.18}$$

with hermitian conjugate:

$$\zeta^+ = [\zeta_1^*, \quad \zeta_2^*] \tag{16.19}$$

We obtain the invariant:

$$X^2 + Y^2 + Z^2 = \zeta_1\zeta_1^* + \zeta_2\zeta_2^* \tag{16.20}$$

and therefore a relation between the Cartesian $O(3)$ elements and the $SU(2)$ elements. The chirality column vector in $SU(2)$ representation is defined as:

$$V^a = \begin{bmatrix} \zeta_1 \\ \zeta_2 \end{bmatrix} = \begin{bmatrix} e^R \\ e^L \end{bmatrix} \tag{16.21}$$

with elements:

$$\zeta_1 = \zeta_2^* = \frac{1}{\sqrt{2}}(1-i) \tag{16.22}$$

The spin column vector in $SU(2)$ representation is defined as:

$$V^\mu = e^{-i\phi} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \tag{16.23}$$

where ϕ is the phase of the fermionic field. The chirality and spin column vectors are related by the $SU(2)$ tetrad field q^a_μ :

$$V^\mu = q^a_\mu V^\mu \tag{16.24}$$

i.e.

$$q^a_\mu = \frac{e^{i\phi}}{\sqrt{2}} \begin{bmatrix} 1 & -i \\ 1 & i \end{bmatrix} \tag{16.25}$$

The tetrad (16.25) is a simple example of a right or left handed spinning field. It can be seen that:

$$\zeta_1\zeta_2 = 1 = \frac{1}{\sqrt{2}}(1-i)\frac{1}{\sqrt{2}}(1+i) \tag{16.26}$$

so the origin of chirality, or left and right handedness, is the factorization in Eq.(16.26). If the phase is defined for a fermionic field propagating for convenience along the Y axis:

$$\phi = \omega t - \kappa Y \tag{16.27}$$

it is seen that:

$$\Box q^a_\mu = 0 \tag{16.28}$$

and this is the equation of the hypothetically massless fermion, the Weyl equation. If the two rows of the tetrad matrix in Eq.(16.25) are transposed into column vectors:

$$\psi = \frac{e^{i\phi}}{\sqrt{2}} \begin{bmatrix} 1 \\ -i \\ 1 \\ i \end{bmatrix} \tag{16.29}$$

The four-spinor ψ consists of two Pauli spinors:

$$\phi^R = \frac{e^{i\phi}}{\sqrt{2}} \begin{bmatrix} 1 \\ -i \end{bmatrix}, \quad \phi^L = \frac{e^{i\phi}}{\sqrt{2}} \begin{bmatrix} 1 \\ i \end{bmatrix} \tag{16.30}$$

and obeys the equation:

$$\Box \psi = 0 \tag{16.31}$$

If the Y Pauli matrix is denoted by:

$$\sigma_Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \tag{16.32}$$

the Weyl equations can be expressed as:

$$\sigma_Y \phi^R = -\phi^R \tag{16.33}$$

$$\sigma_Y \phi^L = -\phi^L \tag{16.34}$$

ie:

$$\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \begin{bmatrix} 1 \\ -i \end{bmatrix} = \begin{bmatrix} -1 \\ i \end{bmatrix} = - \begin{bmatrix} 1 \\ -i \end{bmatrix} \tag{16.35}$$

$$\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \begin{bmatrix} 1 \\ i \end{bmatrix} = \begin{bmatrix} 1 \\ i \end{bmatrix} \tag{16.36}$$

The helicity eigenvalues in Eqs.(16.35) and (16.36) are $\pm$ 1. The eigenfunctions are the right and left Pauli spinors ϕ^R and ϕ^L and the eigen-operator is σ_Y. Therefore the Pauli spinors ϕ^R and ϕ^R are those of a massless fermion propagating along Y. This is a simple example of how chirality or helicity can be built up from two types of column vector, one static and representing the sense of handedness (right or left) of the spin, and the other the spin itself. The resulting tetrad is therefore a combination of right and left spin, and for a phase of the type (16.27), propagates along Y. This is an example in special relativity because the Weyl equations are the massless Dirac equations. In general relativity [1]– [8]:

$$\begin{bmatrix} e^R \\ e^L \end{bmatrix} = \begin{bmatrix} q_1^R & q_2^R \\ q_1^L & q_2^L \end{bmatrix} \begin{bmatrix} q^1 \\ q^2 \end{bmatrix} \tag{16.37}$$

and:

$$(\Box + kT)q_\mu^a = 0 \tag{16.38}$$

where:

$$q_\mu^a = \begin{bmatrix} q_1^R & q_2^R \\ q_1^L & q_2^L \end{bmatrix} \tag{16.39}$$

If we define:

$$\psi = \begin{bmatrix} q_1^R \\ q_2^R \\ q_1^L \\ q_2^L \end{bmatrix} \tag{16.40}$$

then the ECE wave equation is:

$$(\Box + kt)\psi = 0 \tag{16.41}$$

It is known experimentally that this equation must reduce to the Dirac equation for the free single fermion uninfluenced by any other type of field:

$$\left(\Box + \left(\frac{mc}{\hbar}\right)^2\right)\psi = 0 \tag{16.42}$$

In this limit:

$$kT = \frac{m^2c^2}{\hbar^2} \tag{16.43}$$

Eq.(16.43) is an example of the equivalence principle. The effect of gravitation, or any other type of field, or combination of fields, on a fermion is given by Eq.(16.41). Thus Eq.(16.41) describes for example the gravitational interaction between two fermions such as electrons. The electric interaction between two electrons is of course described by the Coulomb Law, which can also be expressed in terms of general relativity using ECE theory [1]– [8]. The electromagnetic field is described in ECE theory by the 4×4 tetrad defined by:

$$V^a = q^a_\mu V^\mu \tag{16.44}$$

where V^a is the chirality vector of electromagnetism, and V^μ its spin vector. In general:

$$a = (0), (1), (2), (3) \tag{16.45}$$

and

$$\mu = 0, X, Y, Z \tag{16.46}$$

The space indices of a are those of the complex circular basis and those of μ are in the Cartesian basis. It was discovered experimentally by Arago in 1811 that the electromagnetic field is left (L) or right (R) circularly polarized. Therefore each index (1) has L and R components and similarly for (2). The (3) index is longitudinal and is similarly defined for L and R. The (0) index is time-like. Therefore the left handed circularly polarized transverse potential field component is:

$$\boldsymbol{A}^{(1)}_L = \frac{A^{(0)}}{\sqrt{2}}(\mathbf{i} - i\mathbf{j})e^{i\phi} \tag{16.47}$$

and the right handed circularly polarized transverse component is:

$$\boldsymbol{A}^{(1)}_R = \frac{A^{(0)}}{\sqrt{2}}(\mathbf{i} + i\mathbf{j})e^{i\phi} \tag{16.48}$$

where ϕ is the electromagnetic phase and where $*$ denotes complex conjugate [1]– [8]. The complex conjugates of (16.47) and (16.48) are found by reversing the sign of i:

$$\boldsymbol{A}^{(2)}_L = \frac{A^{(0)}}{\sqrt{2}}(\mathbf{i} + i\mathbf{j})e^{-i\phi} \tag{16.49}$$

$$\boldsymbol{A}^{(2)}_R = \frac{A^{(0)}}{\sqrt{2}}(\mathbf{i} - i\mathbf{j})e^{-i\phi} \tag{16.50}$$

The complex circular unit vector basis has $O(3)$ symmetry and is:

$$\mathbf{e}^{(1)} \times \mathbf{e}^{(2)} = i\mathbf{e}^{(3)*} \tag{16.51}$$

where:

$$\mathbf{e}^{(1)} = \mathbf{e}^{(2)*} = \frac{1}{\sqrt{2}}(\mathbf{i} - i\mathbf{j}) \tag{16.52}$$

$$\mathbf{e}^{(3)} = \mathbf{k} \tag{16.53}$$

For left circular polarization the chirality and spin column vectors appropriate to the transverse potential vector (16.47) to (16.50) can therefore be defined by:

$$V^{(1)}_L = \frac{1}{\sqrt{2}}\begin{bmatrix} 0 \\ 1 \\ -i \\ 0 \end{bmatrix}, \quad V^\mu = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix} e^{-i\phi} \tag{16.54}$$

and the tetrad is therefore:

$$q^{(1)}_{L\mu} = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -i & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} e^{i\phi} \tag{16.55}$$

with individual scalar components:

$$\begin{aligned} q^{(1)}_{LX} &= \frac{1}{\sqrt{2}} e^{i\phi}, \\ q^{(1)}_{LY} &= \frac{-i}{\sqrt{2}} e^{i\phi}, \end{aligned} \tag{16.56}$$

The complete transverse tetrad vector is:

$$q^{(1)}_{L} = \frac{1}{\sqrt{2}} (\mathbf{i} - i\mathbf{j}) e^{i\phi} \tag{16.57}$$

and from the ansatz (16.2) the electromagnetic potential's transverse vector is:

$$\mathbf{A}^{(1)}_{L} = A^{(0)} \mathbf{q}^{(1)}_{L} = \frac{A^{(0)}}{\sqrt{2}} (\mathbf{i} - i\mathbf{j}) e^{i\phi} \tag{16.58}$$

In right circular polarization:

$$V^{(1)}_{R} = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 \\ 1 \\ i \\ 0 \end{bmatrix}, \quad V^{\mu} = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix} e^{-i\phi} \tag{16.59}$$

giving:

$$q^{(1)}_{R\mu} = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & i & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} e^{i\phi} \tag{16.60}$$

and:

$$q^{(1)}_{R} = \frac{1}{\sqrt{2}} (\mathbf{i} + i\mathbf{j}) e^{i\phi} \tag{16.61}$$

$$\mathbf{A}^{(1)}_{R} = A^{(0)} \mathbf{q}^{(1)}_{L} = \frac{A^{(0)}}{\sqrt{2}} (\mathbf{i} + i\mathbf{j}) e^{i\phi} \tag{16.62}$$

The complex conjugates of index (2) are obtained straightforwardly from these equations of index (1) by reversing the sign of i wherever it occurs. It is seen that:

$$q^{(1)}_{0} = q^{(1)}_{Z} = q^{(2)}_{0} = q^{(2)}_{Z} = 0 \tag{16.63}$$

The chiral and spin vectors in longitudinal polarization are defined by:

$$V^{(3)}_{R} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}, V^{(3)}_{L} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -1 \end{bmatrix}, V^{\mu} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \tag{16.64}$$

and

$$q_Z^{(3)} = \pm 1, A^{(3)} = \pm A^{(0)}\mathbf{k} \tag{16.65}$$

The sign change depends on whether the field is left or right circularly polarized.

The time-like polarizations are given by:

$$V^{(0)} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} = V^{\mu} \tag{16.66}$$

and:

$$q_{\mu}^{(0)} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{16.67}$$

ie.

$$q_0^{(0)} = 1, A_0^{(0)} = A^{(0)} q_0^{(0)} \tag{16.68}$$

and are the same for both senses of polarization.

It can be seen that: $\boldsymbol{A}_R^{(1)}$, $\boldsymbol{A}_L^{(1)}$, $\boldsymbol{A}^{(3)}$ and $\boldsymbol{A}^{(0)}$ are solutions of the ECE wave equation [1]– [8] in the approximation:

$$\left(\Box + \left(\frac{mc}{\hbar}\right)^2\right) A_{\mu}^{a} = 0 \tag{16.69}$$

where m is the mass of the photon. It is known that m is very tiny so to an excellent approximation:

$$\Box A_{\mu}^{a} = 0 \tag{16.70}$$

which is a generally covariant form of the d'Alembert equation. For finite photon mass free of any other field such as gravitation we obtain a generally covariant form of the Proca equation:

$$(\Box + kT)A_{\mu}^{a} = 0, kT = \left(\frac{mc}{\hbar}\right)^2 \tag{16.71}$$

In ECE theory both the d'Alembert and Proca equations of special relativity (the standard model) are made generally covariant as required by the most basic principle of general relativity, the principle of general covariance of any equation of physics. The standard model is not generally covariant and breaks this principle. Another problem occurs in the standard model because there the Proca equation is not gauge invariant [10], and is therefore unphysical, conflicting diametrically with photon mass theory in the standard model. Photon mass theory is at the root of the bending of light by gravity, proven experimentally to an accuracy of 1: 100,000 by NASA Cassini. So the standard model is hopelessly self-inconsistent because in one part of it, Einstein Hilbert theory, the photon mass is proven with this accuracy, and in another part, gauge theory, the photon mass is unphysical because the Proca equation is not gauge invariant. The fault lies with the gauge principle because of its abstract and ad hoc introduction of a fibre bundle by Yang and Mills. In fibre bundle theory there is no geometrical interpretation of the a index, as required by general relativity.

The origin of circular polarization of the electromagnetic field on ECE theory are therefore the chirality (or helicity) vectors $V_L^{(1)}$ and $V_R^{(1)}$. These are four vectors, representing one time-like and three space-like polarization. The electromagnetic potential field is the tetrad field within $A^{(0)}$, and the tetrad links the chirality and spin vectors. For the fermion field the tetrad superimposes the chirality and spin vectors to give the Dirac spinor in the limit of a free fermion. In this case:

$$a = L,\ R \tag{16.72}$$

$$\mu = 1,\ 2 \tag{16.73}$$

but the overall method is the same, indicating that ECE is a unified field theory. Indeed, the electromagnetic field can be described [2] in the same way as the fermion field by defining the chiral and spin vectors of the electromagnetic field by:

$$V^a = \frac{1}{\sqrt{2}} \begin{bmatrix} 1-i \\ 1+i \end{bmatrix},\ V^\mu = \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{-i\phi} \tag{16.74}$$

giving the space-like part of the electromagnetic tetrad:

$$q^a_\mu = \frac{e^{i\phi}}{\sqrt{2}} \begin{bmatrix} 1 & -i \\ 1 & i \end{bmatrix} \tag{16.75}$$

The upper and lower row vectors of the tetrad are transposed to column vectors giving:

$$\psi_{em} = \frac{e^{i\phi}}{\sqrt{2}} \begin{bmatrix} q^{(1)}_{XR} \\ q^{(1)}_{YR} \\ q^{(1)}_{XL} \\ q^{(1)}_{YL} \end{bmatrix} \tag{16.76}$$

This is a column four-vector analogous to the Dirac spinor, and the electromagnetic potential in this massless approximation obeys:

$$\Box A^a_\mu = 0 \tag{16.77}$$

where:

$$A^a_\mu = A^{(0)}\psi_{em} \tag{16.78}$$

It is seen that equation (16.77) is the same for the 2×2 square matrix (16.75) or the column vector (16.76). This shows that the electromagnetic field's transverse components [11] can be put into $SU(2)$ representation as first shown by Majorana in the nineteen twenties.

16.3 The fundamental chiral elements and field unification

The right and left handed spins in the chirality column vector:

$$V^a = \frac{1}{\sqrt{2}} \begin{bmatrix} 1-i \\ 1+i \end{bmatrix} \tag{16.79}$$

are the most fundamental elements of the fermion field. The well known Pauli spinors are constructed from the tetrad as follows:

$$(\phi^R)^T = [1\ 0] q^a_\mu \tag{16.80}$$

$$(\phi^L)^T = [0\ 1] q^a_\mu \tag{16.81}$$

where the superscript T denotes "transpose" [12]. Thus:

$$(\phi^R)^T = [1\ 0] \begin{bmatrix} q_1^R & q_2^R \\ q_1^L & q_2^L \end{bmatrix} = \begin{bmatrix} q_1^R & q_2^R \end{bmatrix} \tag{16.82}$$

and:

$$\phi^R = \begin{bmatrix} q_1^R \\ q_2^R \end{bmatrix} \tag{16.83}$$

Similarly:

$$(\phi^L)^T = [0\ 1] \begin{bmatrix} q_1^R & q_2^R \\ q_1^L & q_2^L \end{bmatrix} = \begin{bmatrix} q_1^L & q_2^L \end{bmatrix} \tag{16.84}$$

and:

$$\phi^L = \begin{bmatrix} q_1^L \\ q_2^L \end{bmatrix} \tag{16.85}$$

Thus:

$$\phi^R = (\begin{bmatrix} 1, 0 \end{bmatrix} q^a_\mu)^T \tag{16.86}$$

$$\phi^L = (\begin{bmatrix} 0, 1 \end{bmatrix} q^a_\mu)^T \tag{16.87}$$

and the Dirac spinor is:

$$\psi = \begin{bmatrix} \phi^R \\ \phi^L \end{bmatrix} \tag{16.88}$$

It follows from the ECE equation:

$$(\Box + kT) q^a_\mu = 0 \tag{16.89}$$

that:

$$(\Box + kT)\psi = 0 \tag{16.90}$$

For a free fermion unaffected by any other type of field:

$$kT = \left(\frac{mc}{\hbar}\right)^2 \tag{16.91}$$

and we recover the Dirac equation [1]– [8]:

$$\left(\Box + \left(\frac{mc}{\hbar}\right)^2\right)\psi = 0 \tag{16.92}$$

or

$$\left(\Box + \left(\frac{mc}{\hbar}\right)^2\right) q^a_\mu = 0 \tag{16.93}$$

It is seen that the most fundamental elements of ψ are V^a and V^μ, and that generally covariant Dirac equation is:

$$(\Box + kT)\psi = 0 \tag{16.94}$$

where:

$$\psi = \begin{bmatrix} q^R \\ q^L \end{bmatrix} = \begin{bmatrix} q_1^R \\ q_2^R \\ q_1^L \\ q_2^L \end{bmatrix} \tag{16.95}$$

The tetrad is therefore:

$$q_\mu^a = \begin{bmatrix} (\phi^R)^T \\ (\phi^L)^T \end{bmatrix} \tag{16.96}$$

and the Dirac spinor is:

$$\psi = \begin{bmatrix} \phi^R \\ \phi^L \end{bmatrix} \tag{16.97}$$

For Eqs.(16.82) and (16.84):

$$q_\mu^a = \begin{bmatrix} 1 \\ 0 \end{bmatrix} (\phi^R)^T + \begin{bmatrix} 0 \\ 1 \end{bmatrix} (\phi^L)^T \tag{16.98}$$

i.e.:

$$\begin{aligned} \begin{bmatrix} q_1^R & q_2^R \\ q_1^L & q_2^L \end{bmatrix} &= \begin{bmatrix} 1 \\ 0 \end{bmatrix} \begin{bmatrix} q_1^R & q_2^R \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} q_1^L & q_2^L \end{bmatrix} \\ &= \begin{bmatrix} q_1^R & q_2^R \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ q_1^L & q_2^L \end{bmatrix} \end{aligned} \tag{16.99}$$

Eq.(16.98) shows that the tetrad can be analyzed as te sum of two transposed Pauli spinors. Therefore the fundamental Pauli spinors are made up of elements of Cartan geometry, the chirality column vector V^a and the spin column vector V^μ.

Now define the state spinors:

$$\zeta_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad \zeta_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \tag{16.100}$$

which are inter-convertible by the parity inversion operator $\hat{P}$:

$$\hat{P}\zeta_1 = \zeta_2 \tag{16.101}$$

and so may be considered as fundamental states of handedness or chirality. The analysis has been carried out for the fermion field with $SU(2)$ symmetry. The $SU(2)$ group is the unitary unimodular group of matrices such as:

$$S = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & -i \\ -i & 1 \end{bmatrix} \tag{16.102}$$

whose hermitian transpose is:

$$S^+ = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & i \\ i & 1 \end{bmatrix} \tag{16.103}$$

so that:

$$SS^+ = \frac{1}{2} \begin{bmatrix} 1 & -i \\ -i & 1 \end{bmatrix} \begin{bmatrix} 1 & i \\ i & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{16.104}$$

and $det\, S = 1$. Eq.(16.79) can be developed into the sum:

$$V^a = \frac{1}{\sqrt{2}}\left(\begin{bmatrix} 1 \\ 1 \end{bmatrix} - i(\zeta_1 - \zeta_2)\right) \tag{16.105}$$

and so the chirality vector is made up of the real part:

$$Re\, V^a = \frac{1}{\sqrt{2}}\mathbf{1} := \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 1 \end{bmatrix} \tag{16.106}$$

and the imaginary part:

$$Im\, V^a = \frac{1}{\sqrt{2}}(\zeta_2 - \zeta_1) \tag{16.107}$$

These can be regarded as the fundamental elements of chirality or handedness.

The unitary unimodular matrix S is made up of two Pauli matrices:

$$S = \frac{1}{\sqrt{2}}(\sigma_0 - i\sigma_1) \tag{16.108}$$

where:

$$\sigma_0 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad \sigma_1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \tag{16.109}$$

The other two Pauli matrices are:

$$\sigma_2 = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}, \quad \sigma_3 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \tag{16.110}$$

The related matrix:

$$\zeta = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \tag{16.111}$$

has the orthoganility [12] property:

$$\zeta\zeta^T = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{16.112}$$

and so:

$$S = \frac{1}{\sqrt{2}}(\mathbf{1} + i(\zeta_2 - \zeta_1)) \tag{16.113}$$

is unitary and unimodular [12] while:

$$\zeta = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & \zeta_2 - \zeta_1 \\ 1 & \end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix} \mathbf{1} & \zeta_2 - \zeta_1 \end{bmatrix} \tag{16.114}$$

is orthogonal. The $SU(2)$ matrix:

$$S = \begin{bmatrix} q_X^{(1)} & q_Y^{(1)} \\ -iq_X^{(1)} & -iq_Y^{(1)} \end{bmatrix} e^{-i\phi} \tag{16.115}$$

where:

$$q_X^{(1)} = \frac{1}{\sqrt{2}}e^{i\phi}, \quad q_Y^{(1)} = \frac{-i}{\sqrt{2}}e^{i\phi} \tag{16.116}$$

and

$$\boldsymbol{A}^{(1)} = A^{(1)}\boldsymbol{q}^{(1)} = A^{(0)}(q_X^{(1)}\boldsymbol{i} + q_Y^{(1)}\boldsymbol{j}) \tag{16.117}$$

give and $SU(2)$ representation of the electromagnetic field. Using the further development:

$$\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \tag{16.118}$$

it is found that the chirality vector is made up of the elements ζ_1 and ζ_2 as follows:

$$V^a = \frac{1}{\sqrt{2}}(\zeta_1 + \zeta_2 - i(\zeta_1 - \zeta_2)) \tag{16.119}$$

and that the spin vector is made up of the same elements as follows:

$$V^\mu = (\zeta_1 + \zeta_2)e^{-i\phi} \tag{16.120}$$

So it is concluded that the fermion field's most fundamental elements are:

$$\zeta_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad \zeta_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \tag{16.121}$$

together with the phase factor $e^{-i\phi}$. We define ζ_1 and ζ_2 as the chiral elements of the field. These elements also define the electromagnetic field's transverse elements as follows:

$$V_L^{(1)} = \frac{1}{\sqrt{2}}\left(\begin{bmatrix} 0 \\ \zeta_1 \\ 0 \end{bmatrix} - i\begin{bmatrix} 0 \\ \zeta_2 \\ 0 \end{bmatrix}\right) \tag{16.122}$$

$$V^\mu = \frac{1}{\sqrt{2}}\left(\begin{bmatrix} 0 \\ \zeta_1 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ \zeta_2 \\ 0 \end{bmatrix}\right)e^{-i\phi} \tag{16.123}$$

and are therefore unifying elements of the fermionic and electro-magnetic fields. The tetrad elements of the electromagnetic field can be put in $SU(2)$ form by using:

$$\boldsymbol{q}^{(1)} \cdot \boldsymbol{q}^{(2)} + \boldsymbol{q}^{(2)} \cdot \boldsymbol{q}^{(1)} + \boldsymbol{q}^{(3)} \cdot \boldsymbol{q}^{(3)} = \boldsymbol{q}_1\boldsymbol{q}_1^* + \boldsymbol{q}_2\boldsymbol{q}_2^* \tag{16.124}$$

For circular polarization:

$$\begin{aligned} \boldsymbol{q}^{(1)} &= \frac{1}{\sqrt{2}}(\boldsymbol{i} - i\boldsymbol{j})e^{i\phi} = \boldsymbol{q}^{(2)*}, \\ \boldsymbol{q}^{(3)} &= \boldsymbol{k} \end{aligned} \tag{16.125}$$

and so:

$$q_1q_1^* + q_2q_2^* = 3 \tag{16.126}$$

A possible solution is:

$$q_1 = \sqrt{\frac{3}{2}}e^{i\phi}, \quad q_2 = -\sqrt{\frac{3}{2}}e^{i\phi} \tag{16.127}$$

Geometrically, the electromagnetic field has the same origin as the fermionic field, as shown by Eq.(16.124).

Finally in this section the minimal prescription as used for example by Dirac to define the well known half integral spin of the fermion in ESR or NMR is defined for use in ECE theory. The ECE Lemma is [1]– [8]

$$\Box q^a_\mu = R q^a_\mu \tag{16.128}$$

where R is a well defined [1]– [8] scalar curvature and where:

$$\Box = \partial_\mu \partial^\mu = \gamma^\mu \gamma^\nu \partial_\mu \partial_\nu \tag{16.129}$$

Here γ^μ is a Dirac matrix. The d'Alembertian operator has been isolated in Eq.(16.128) from an analysis [1]– [8] of the tetrad postulate. The minimal prescription is a momentum addition which describes the effect of the electromagnetic field on the fermion field in Dirac's original analysis [1]– [8]. In gauge theory it is the result of the gauge invariance principle [9], but that is an abstract procedure which as we have argued already, leads to a diametric self inconsistency in the standard model. In ECE theory (general relativity) the minimal prescription is developed as in the original intent - a momentum addition.

In ECE the quantum operator equivalence is used in the same form as special relativity, because the partial four-derivative in ECE is the same as in special relativity [1]– [9]:

$$p^\mu = i\hbar \partial^\mu \tag{16.130}$$

where:

$$p^\mu = \left(\frac{E_n}{c}, \boldsymbol{p} \right), \quad \partial^\mu = \left(\frac{1}{c}\frac{\partial}{\partial t}, -\boldsymbol{\nabla} \right) \tag{16.131}$$

as usual. Now define:

$$A_\mu := A^{(0)}_\mu + A^{(1)}_\mu + A^{(2)}_\mu + A^{(3)}_\mu \tag{16.132}$$

so that all four states of polarization of A^a_μ are accounted for. The semi-classical minimal prescription in ECE theory is then defined as:

$$p_\mu \rightarrow p_\mu + eA_\mu \tag{16.133}$$

$$p^\mu \rightarrow p^\mu + eA^\mu \tag{16.134}$$

This means that:

$$\partial_\mu \rightarrow \partial_\mu - i\frac{e}{\hbar}A_\mu \tag{16.135}$$

$$\partial^\mu \rightarrow \partial^\mu - i\frac{e}{\hbar}A^\mu \tag{16.136}$$

and

$$\Box = \partial_\mu \partial^\mu \rightarrow \left(\partial_\mu - i\frac{e}{\hbar}A_\mu \right) \left(\partial^\mu - i\frac{e}{\hbar}A^\mu \right) \tag{16.137}$$

i.e.

$$\Box' := \Box - i\frac{e}{\hbar}\left(A_\mu \partial^\mu + A^\mu \partial_\mu \right) - i\frac{e^2}{\hbar^2}A_\mu A^\mu \tag{16.138}$$

The wave equation that defines the interaction of the Dirac spinor ψ and A_μ is therefore

$$(\Box' + kT)\psi = 0 \tag{16.139}$$

In the absence of gravitation, this becomes:

$$(\Box' + \frac{m_e c}{\hbar})\psi = 0 \tag{16.140}$$

Eq.(16.140) produces all the familiar half integral spin magnetic effects such as the Stern Gerlach experiment, the Zeeman effect, ESR, NMR and MRI, and for the electromagnetic field, RFR [1]– [8]. The overall effect in these phenomena is described by:

$$\Box \rightarrow \Box' \tag{16.141}$$

Any type of field interaction can be described in this way. If for example we wish to describe the effect of gravitation on interacting electromagnetic and fermionic fields Eq.(16.139) must be used. Both the fermion and electromagnetic fields in this case are affected by gravitation through the Cartan geometry defined in Eqs.(16.7) to (16.10). The gravitational field is represented by the Riemann or curvature form. The fermion field in these equations is the tetrad q^a_μ in $SU(2)$ representation, a 2×2 matrix. If Eqs.(16.7) to (16.10) are then developed in $SU(2)$ representation there is an $SU(2)$ symmetry torsion and curvature which interact with each other. The interaction of the fermion and gravitational fields is then governed in this way - again by Cartan geometry. The minimal prescription used in Eq.(16.139) is semi-classical - the fermion field is quantized but the electromagnetic field is classical. The next level is a fully quantized theory in which the electromagnetic field is governed by the ECE wave equation:

$$(\Box + kT)A^a_\mu = 0 \tag{16.142}$$

From Eqs.(16.133) and (16.134) and conservation of momentum, the photon momentum is changed by the electron momentum and vice versa in such a way that the total momentum (photon plus electron) before and after collision is the same but the individual momenta of photon and electron are changed. If the electron momentum is for example increased by a collision, the photon momentum must be decreased as follows [1]:

$$A_\mu \rightarrow A_\mu - \frac{1}{e}p_\mu \tag{16.143}$$

Thus Eq.(16.142) is changed to:

$$(\Box'' + kT)A^a_\mu = 0 \tag{16.144}$$

where:

$$\Box'' = \Box + \frac{i}{\hbar}(p_\mu \partial^\mu + p^\mu \partial_\mu) - \frac{p_\mu p^\mu}{\hbar^2} \tag{16.145}$$

If there is no gravitation present:

$$kT = \left(\frac{m_p c}{\hbar}\right)^2 \tag{16.146}$$

where m_p is the photon mass. Therefore the problem is to solve simultaneously the following equations:

$$\left(\Box' + \left(\frac{m_p c}{\hbar}\right)^2\right)\psi = 0 \tag{16.147}$$

and

$$\left(\Box'' + \left(\frac{m_p c}{\hbar}\right)^2\right) A^a_\mu = 0 \tag{16.148}$$

The electron mass m_e is many orders of magnitude greater than the photon mass, perhaps as many as forty orders of magnitude greater. Therefore in most textbook treatments of for example the Compton effect, the energy of the electron is represented in the classical special relativistic limit by the Einstein equation:

$$En = (m^2c^4 + p^2c^2)^{\frac{1}{2}} \tag{16.149}$$

but the photon is represented as a pure wave with no mass using the de Broglie equation:

$$En = hv, \quad p = \frac{hv}{c} \tag{16.150}$$

In Feynman's quantum electrodynamics, exchange of a virtual photon is used. However, in general relativity Eqs.(16.147) and (16.148) must be used and solved simultaneously with sufficient numerical precision to give the experimentally known effects of quantum corrections, such as the anomalous magnetic moment of the electron in the Lamb shift, and the Casimir effect.

Acknowledgements The British Government is thanked for the award of a Civil List pension for distinguished services to science, and the staff of AIAS are thanked for many interesting discussions.

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