Chapter 2

Geodesics and the Aharonov Bohm Effects in ECE Theory

(Paper 56)
by
Myron W. Evans,
Alpha Institute for Advanced Study (AIAS).
(emyrone@aol.com, www.aias.us, www.atomicprecision.com)

Abstract

Geodesics are considered in the Einstein Cartan Evans (ECE) unified field theory. The concept of parallel transport along any curve is extended to the covariant exterior derivative of Cartan geometry. The Lorentz force is thus recognized as the covariant exterior derivative of the potential form along any path in ECE space-time. The class of Aharonov Bohm effects are due to parallel transport of the exterior covariant derivative. The equations of the electromagnetic and gravitational Aharonov Bohm effects are developed in ECE theory.

Keywords: Einstein Cartan Evans (ECE) unified field theory, geodesics, Lorentz force equation, Aharonov Bohm effects.

2.1 Introduction

In Einsteinian philosophy every equation of physics must be rigorously objective to all observers. This is the most fundamental principle of relativity theory, the latter must be generally covariant, covariant under the general coordinate transformation. This principle must apply both on the classical and quantum levels. Recently [1]– [15], a generally covariant unified field theory has been developed which meets these fundamental philosophical requirements, and is known as Einstein Cartan Evans (ECE) field theory. It has been applied to many aspects of physics [1] [2] and has been tested extensively against experimental data. It

also meets the fundamental philosophical requirement known as Ockham's Razor, that a theory of physics must be as simple as possible. In this respect the ECE theory needs only the four dimensions of the original theory of relativity, and is preferred to string theory. The latter uses many unphysical dimensions as is well known. ECE theory is preferred to gauge theory because the latter is not generally covariant in three of its sectors: the electromagnetic, weak and strong sectors. Gauge theory in these sectors superimposes an abstract set of numbers on a Minkowski spacetime, while ECE theory is rigorously covariant in all sectors [1]– [15], being based directly on standard Cartan geometry. Gauge theory runs into trouble in trying to explain the well known Aharonov Bohm (AB) effects, while ECE theory explains them straightforwardly. The standard model runs into trouble in quantum mechanics, because of the lack of objectivity in its quantum mechanical sector, while ECE theory rigorously retains objectivity on the classical and quantum levels. Therefore ECE theory is preferred to the standard model, and explains data in a generally covariant manner under all circumstances.

In Section 2.2 the theory of geodesics in ECE theory is developed by extending the concept of parallel transport to the covariant exterior derivative of Cartan geometry. It is shown that the potential form [1]– [15] is parallel transported along any curve in ECE theory. It follows that the Cartan structure equations are also parallel transported along any curve, and that the Lorentz force equation is the covariant exterior derivative of the potential form along any path in ECE space-time. All AB effects are due to parallel transport of the exterior covariant derivative. These concepts extend the concept of the geodesic in relativity theory. The geodesic is the path that parallel transports it own tangent vector [15] and in gravitational relativity (Einstein Hilbert (EH) field theory) is the path followed by unaccelerated particles. In Section 2.3 the equations of the electromagnetic AB effect are given, and in Section 2.4, the equations are given of the gravitational AB effect.

2.2 Parallel transport and geodesics in ECE Theory, applications to the Lorentz Force equation and Aharonov Bohm effects

In standard general relativity (EH theory [15]) the geodesic is defined as the path that parallel transports its own tangent vector. If $x^{\mu}(\lambda)$ is any curve, the tangent vector is defined [15] as:

$$t^{\mu} = \frac{dx^{\mu}}{d\lambda} \tag{2.1}$$

The covariant derivative along the path is defined [15] as:

$$\frac{D}{d\lambda} := \frac{dx^{\mu}}{d\lambda} D_{\mu} \tag{2.2}$$

where D_{μ} is the covariant derivative. The parallel transport of any tensor is then defined [15] as:

$$\left(\frac{DT}{d\lambda} \right)^{\mu_1 \mu_2 \cdot \mu_k}_{\nu_1 \nu_2 \cdot \mu_l} = \frac{dx^{\sigma}}{d\lambda} D_{\sigma} T^{\mu_1 \mu_2 \cdot \mu_k}_{\nu_1 \nu_2 \cdot \nu_l} = 0 \tag{2.3}$$

In EH theory the connection is always metric compatible [15]:

$$D_\mu g_{\rho\nu} = 0 \tag{2.4}$$

where $g_{\rho\nu}$ is the symmetric metric of EH field theory. Such a connection is always parallel transported [15]:

$$\frac{Dg_{\mu\nu}}{d\lambda} = \frac{dx^\sigma}{d\lambda} D_\sigma g_{\mu\nu} = 0 \tag{2.5}$$

The equation of the geodesic in EH theory is [15]:

$$\frac{D}{d\lambda}\left(\frac{dx^\mu}{d\lambda}\right) = 0 \tag{2.6}$$

and parallel transports the tangent vector. Eq.(2.6) can be rewritten as:

$$\frac{d^2x^\mu}{d\lambda^2} + \Gamma^\mu{}_{\rho\sigma}\frac{dx^\rho}{d\lambda}\frac{dx^\sigma}{d\lambda} = 0 \tag{2.7}$$

which is the best known form of the geodesic equation of EH field theory. The geodesic in EH field theory is the path followed by unaccelerated particles. In flat space-time the connection vanishes, and Eq.(2.7) becomes:

$$\frac{d^2x^\mu}{d\lambda^2} = 0 \tag{2.8}$$

which is the equation of a straight line. Newton's first law is therefore regained, unaccelerated particles move in a straight line. The parameter λ can be related [15] to the proper time τ by:

$$\lambda = a\tau + b \tag{2.9}$$

An affine parameter is related to the proper time in this way. If:

$$\lambda = \tau \tag{2.10}$$

the geodesic equation becomes:

$$\frac{d^2x^\mu}{d\tau^2} + \Gamma^\mu{}_{\rho\sigma}\frac{dx^\rho}{d\tau}\frac{dx^\sigma}{d\tau} = 0 \tag{2.11}$$

and in the Newtonian limit this equation means that:

$$\mathbf{a} = \mathbf{0} \tag{2.12}$$

where $\mathbf{a}$ denotes acceleration. Therefore in the Newtonian limit the geodesic equation is the Newton force law for:

$$\mathbf{f} = m\mathbf{a} = \mathbf{0} \tag{2.13}$$

The Newton force law is therefore the limit of a more general geometry than Euclidean geometry. For EH theory this is well known to be Riemann geometry. For ECE theory Riemann geometry is generalized to the well known Cartan geometry [1]– [15].

When there is a force present the right hand side of Eq.(2.11) is no longer zero. The standard model Lorentz force for example [15] is:

$$f^{\mu} = eU^{\lambda}F_{\lambda}{}^{\mu} = eF^{\mu}{}_{\nu}\frac{dx^{\nu}}{d\tau} \tag{2.14}$$

where $F^{\mu}{}_{\nu}$ is the standard model electromagnetic field tensor and U^{λ} is the four velocity. In the presence of a Lorentz force, the electron no longer moves along a geodesic. The standard model Lorentz force, however, is Lorentz covariant, and is therefore the result of a theory of special relativity, not of general relativity as required. In this section the required generally covariant Lorentz force equation is developed with ECE theory. In so doing the effect of gravitation on the Lorentz force may be estimated. This is not possible in the standard model.

Parallel transport is defined for the arbitrary tensor T (indices suppressed for clarity) as:

$$\frac{dT}{d\lambda} = \frac{dx^{\mu}}{d\lambda}\frac{\partial T}{\partial x^{\mu}} = 0 \tag{2.15}$$

This result comes from the rule for differentiation [16] of a function of a function. If:

$$T = T\left(x^{\mu}\left(\lambda\right)\right) \tag{2.16}$$

then:

$$\frac{dT}{d\lambda} = \frac{dT}{dx^{\mu}}\frac{dx^{\mu}}{d\lambda} \tag{2.17}$$

The rule (2.17) may be extended to the covariant derivative of a tensor of any rank:

$$\frac{DT}{d\lambda} = \frac{dx^{\mu}}{d\lambda}D_{\mu}T \tag{2.18}$$

where T is defined by Eq.(2.16). The rule (2.18) is extended in this Section to the covariant exterior derivative of Cartan [1]– [15], used in the first and second Cartan structure equations:

$$T^{a} = D \wedge q^{a} = d \wedge q^{a} + \omega^{a}{}_{b} \wedge q^{b} \tag{2.19}$$

and

$$R^{a}{}_{b} = D \wedge \omega^{a}{}_{b} + \omega^{a}{}_{c} \wedge \omega^{c}{}_{b} \tag{2.20}$$

Here T^{a} is the torsion form, $R^{a}{}_{b}$ is the Riemann or curvature form, q^{a} is the tetrad form, $\omega^{a}{}_{b}$ is the spin connection, $D\wedge$ denotes the covariant exterior derivative and $d\wedge$ denotes the exterior derivative. It will be proven that:

$$\frac{dx^{\mu}}{d\lambda}\left(D \wedge A\right)_{\mu\nu} = \frac{dx^{\mu}}{d\lambda}F^{a}{}_{\mu\nu} \tag{2.21}$$

is the origin of the generally covariant Lorentz force and also the origin of the class of AB effects. Here $F^{a}{}_{\mu\nu}$ is the generally covariant electromagnetic field tensor of ECE theory, defined in standard notation [1]– [15] as:

$$F^{a} = D \wedge A^{a} \tag{2.22}$$

Consider:

$$A^{a}{}_{\mu} = A^{a}{}_{\mu}\left(x^{\nu}\left(\lambda\right)\right) \tag{2.23}$$

and

$$A^{a}{}_{\nu} = A^{a}{}_{\nu}\left(x^{\mu}\left(\lambda\right)\right) \tag{2.24}$$

with:

$$F^{a}{}_{\mu\nu} = \partial_{\mu}A^{a}{}_{\nu} - \partial_{\nu}A^{a}{}_{\mu} + \omega^{a}{}_{\mu b}A^{b}{}_{\nu} - \omega^{a}{}_{\nu b}A^{b}{}_{\mu} \tag{2.25}$$

Then:

$$\frac{DA^{a}{}_{\mu}}{d\lambda} = \frac{dx^{\mu}}{d\lambda}D_{\nu}A^{a}{}_{\mu} \tag{2.26}$$

$$\frac{DA^{a}{}_{\nu}}{d\lambda} = \frac{dx^{\nu}}{d\lambda}D_{\mu}A^{a}{}_{\nu} \tag{2.27}$$

Using the tetrad postulate [1]– [15]:

$$D_{\nu}A^{a}{}_{\mu} = D_{\mu}A^{a}{}_{\nu} = 0, \quad A^{a}{}_{\mu} = A^{(0)}q^{a}{}_{\mu} \tag{2.28}$$

it follows that:

$$\frac{DA^{a}{}_{\mu}}{d\lambda} = \frac{DA^{a}{}_{\nu}}{d\lambda} = 0 \tag{2.29}$$

a result which means that

$$A^{a}{}_{\mu} = A^{(0)}q^{a}{}_{\mu} \tag{2.30}$$

is parallel transported along any curve $x^{\mu}\left(\lambda\right)$. It follows that:

$$\frac{dx^{\mu}}{d\lambda}\left(D_{\nu}A^{a}{}_{\mu} - D_{\mu}A^{a}{}_{\nu}\right) = 0 \tag{2.31}$$

The exterior covariant derivative is defined as:

$$\left(D \wedge A\right)^{a}_{\mu\nu} = F^{a}{}_{\mu\nu} \tag{2.32}$$

with:

$$D_{\mu}A^{a}{}_{\nu} - D_{\nu}A^{a}{}_{\mu} = \partial_{\mu}A^{a}{}_{\nu} - \partial_{\nu}A^{a}{}_{\mu} + \omega^{a}{}_{\mu b}A^{b}{}_{\nu} - \omega^{a}{}_{\nu b}A^{b}{}_{\mu} - F^{a}{}_{\mu\nu} = 0 \tag{2.33}$$

It follows from Eqs.(2.33) and (2.35) that the Cartan structure equation is parallel transported along any curve $x^{\mu}\left(\lambda\right)$:

$$\frac{dx^{\mu}}{d\lambda}\left(F^{a} - D \wedge A^{a}\right)_{\mu\nu} = 0 \tag{2.34}$$

and so:

$$\frac{dx^{\mu}}{d\lambda}\left(D \wedge A^{a}\right)_{\mu\nu} = \frac{dx^{\mu}}{d\lambda}F^{a}{}_{\mu\nu} \tag{2.35}$$

Q.E.D. The generally covariant Lorentz force is the right hand side of Eq. (2.35) multiplied by e, the charge on the electron.

In the absence of the Lorentz force and thus in the absence of acceleration:

$$\frac{dx^{\mu}}{d\lambda}F^{a}{}_{\mu\nu} = \frac{dx^{\mu}}{d\lambda}\left(D \wedge A^{a}\right)_{\mu\nu} = 0 \tag{2.36}$$

and the exterior covariant derivative of Cartan [1]– [15] is parallel transported along any curve.

Similarly:

$$\frac{dx^{\mu}}{d\lambda}R^{a}{}_{b\mu\nu} = \frac{dx^{\mu}}{d\lambda}\left(D \wedge \omega^{a}{}_{b}\right)_{\mu\nu} \tag{2.37}$$

and in the absence of force, the exterior covariant derivative of the spin connection is parallel transported along any curve:

$$\frac{dx^{\mu}}{d\lambda}R^{a}{}_{b\mu\nu} = \frac{dx^{\mu}}{d\lambda}\left(D \wedge \omega^{a}{}_{b}\right)_{\mu\nu} = 0 \tag{2.38}$$

The class of AB effects may now be defined in terms of these equations. The electromagnetic AB effects are defined by Eq.(2.36). When there is no field there may still be a potential present in ECE theory, a potential defined by:

$$A^{(0)}\left(d \wedge q^{a} + \omega^{a}{}_{b} \wedge q^{b}\right) = 0 \tag{2.39}$$

where $cA^{(0)}$ is a primordial voltage. Eq.(2.39) is true for all $A^{(0)}$ and all paths $x^{\mu}(\lambda)$. The Cartan geometry of the electromagnetic AB effects is therefore always:

$$D \wedge q^{a} = 0 \tag{2.40}$$

Similarly the Cartan geometry of the gravitational AB effects is always:

$$D \wedge \omega^{a}{}_{b} = d \wedge \omega^{a}{}_{b} + \omega^{a}{}_{c} \wedge \omega^{c}{}_{b} = 0 \tag{2.41}$$

All AB effects are therefore due to parallel transport of the exterior covariant derivative of Cartan geometry. They are effects of a generally covariant unified field theory [1] [2], of spinning and curving space-time. In the standard model there are no AB effects because in the standard model [1] [2]:

$$F_{\mu\nu} = \left(d \wedge A\right)_{\mu\nu} \tag{2.42}$$

and if F is zero so is A and vice-versa.

2.3 The equations of the electromagnetic AB effects

The class of electromagnetic AB effects is defined by:

$$A^{(0)}D \wedge q = 0 \tag{2.43}$$

where $cA^{(0)}$ is a primordial voltage which is always non-zero. Eq.(2.43) means that when the electromagnetic field is zero the electromagnetic potential is non-zero. In the well known Chambers experiment [1] [2] the magnetic field of an iron whisker does not interact with the electron beams, but there is present a potential which causes a fringe shift in a Young interferometer. Eq.(2.43) is written in the standard notation [1]– [15] of differential geometry as:

$$d \wedge A^{a} + \omega^{a}{}_{b} \wedge A^{b} = 0 \tag{2.44}$$

and implies:

$$F^{a} = 0 \tag{2.45}$$

The fringe shift of the Chambers experiment is described [1] [2] in the minimal prescription by the following shift in the momentum tetrad:

$$p^{a} \rightarrow p^{a} + eA^{a} \tag{2.46}$$

In the standard model:

$$F = d \wedge A \tag{2.47}$$

so if F is zero so is A and vice-versa. In ECE theory A^a can be non-zero if F^a is zero, as first observed by Chambers and in many experiments since then. The mathematical task is to solve Eq.(2.44) for the potential components responsible for the class of electromagnetic AB effects. If we consider electromagnetism free from gravitational effects [1]– [14], then the spin connection is dual to the tetrad as follows:

$$\omega^a{}_b = -\frac{\kappa}{2}\epsilon^a{}_{bc}q^c \tag{2.48}$$

Here:

$$\epsilon^a{}_{bc} = \eta^{ad}\epsilon_{dbc} \tag{2.49}$$

where

$$\begin{aligned}\eta^{ad} &= \mathrm{diag}\,(1,-1,-1,-1) \\ &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}\end{aligned} \tag{2.50}$$

is the Minkowski metric of the tangent space-time at point P to the base manifold of ECE theory [1]– [14]. In Eq.(2.49) ϵ_{abc} is the three index, totally antisymmetric unit tensor in four dimensions. This is defined in general as follows:

$$\left.\begin{array}{c} \epsilon_{123} = \epsilon_{231} = \epsilon_{312} = 1, \\ \epsilon_{132} = \epsilon_{213} = \epsilon_{321} = -1, \\ \epsilon_{012} = \epsilon_{120} = \epsilon_{201} = 1, \\ \epsilon_{021} = \epsilon_{102} = \epsilon_{210} = -1, \\ \epsilon_{012} = \epsilon_{301} = \epsilon_{130} = \epsilon_{023} = \epsilon_{302} = \epsilon_{230} = 1, \\ \epsilon_{031} = \epsilon_{310} = \epsilon_{103} = \epsilon_{032} = \epsilon_{320} = \epsilon_{203} = -1 \end{array}\right\} \tag{2.51}$$

Therefore:

$$\left.\begin{aligned}
\epsilon^{1}{}_{23} &= g^{1d}\epsilon_{d23} = g^{11}\epsilon_{123} = -\epsilon_{123},\\
\epsilon^{3}{}_{12} &= g^{3d}\epsilon_{d12} = g^{33}\epsilon_{312} = -\epsilon_{312},\\
\epsilon^{2}{}_{31} &= g^{2d}\epsilon_{d31} = g^{22}\epsilon_{231} = -\epsilon_{231},\\
\epsilon^{0}{}_{12} &= g^{00}\epsilon_{012} = \epsilon_{012},\\
\epsilon^{2}{}_{01} &= g^{22}\epsilon_{201} = -\epsilon_{120},\\
\epsilon^{0}{}_{13} &= g^{00}\epsilon_{013} = \epsilon_{013},\\
\epsilon^{0}{}_{23} &= g^{00}\epsilon_{023} = \epsilon_{023},\\
\epsilon^{3}{}_{01} &= g^{33}\epsilon_{301} = \epsilon_{301},\\
\epsilon^{2}{}_{30} &= g^{22}\epsilon_{230} = -\epsilon_{230}.
\end{aligned}\right\} \tag{2.52}$$

Eq.(2.44) means that:

$$\left.\begin{aligned}
d\wedge A^0 + \omega^0{}_b \wedge A^b = 0,\\
d\wedge A^1 + \omega^1{}_b \wedge A^b = 0,\\
d\wedge A^2 + \omega^2{}_b \wedge A^b = 0,\\
d\wedge A^3 + \omega^3{}_b \wedge A^b = 0,
\end{aligned}\right\} \tag{2.53}$$

with summation implied over repeated contravariant-covariant indices as follows:

$$\left.\begin{aligned}
d\wedge A^0 + \omega^0{}_1 \wedge A^1 + \omega^0{}_2 \wedge A^2 + \omega^0{}_3 \wedge A^3 = 0,\\
d\wedge A^1 + \omega^1{}_0 \wedge A^0 + \omega^1{}_2 \wedge A^2 + \omega^1{}_3 \wedge A^3 = 0,\\
d\wedge A^2 + \omega^2{}_0 \wedge A^0 + \omega^2{}_1 \wedge A^1 + \omega^2{}_3 \wedge A^3 = 0,\\
d\wedge A^3 + \omega^3{}_0 \wedge A^0 + \omega^3{}_1 \wedge A^1 + \omega^3{}_2 \wedge A^2 = 0
\end{aligned}\right\} \tag{2.54}$$

The various spin connections are therefore:

$$\left.\begin{aligned}
\omega^0{}_1 &= -\frac{\kappa}{2}\left(\epsilon^0{}_{12}q^2 + \epsilon^0{}_{13}q^3\right), \\
\omega^0{}_2 &= -\frac{\kappa}{2}\left(\epsilon^0{}_{23}q^3 + \epsilon^0{}_{21}q^1\right), \\
\omega^0{}_3 &= -\frac{\kappa}{2}\left(\epsilon^0{}_{31}q^1 + \epsilon^0{}_{32}q^2\right), \\
\omega^1{}_2 &= -\frac{\kappa}{2}\left(\epsilon^1{}_{20}q^0 + \epsilon^1{}_{23}q^3\right), \\
\omega^1{}_3 &= -\frac{\kappa}{2}\left(\epsilon^1{}_{32}q^2 + \epsilon^1{}_{30}q^0\right), \\
\omega^2{}_3 &= -\frac{\kappa}{2}\left(\epsilon^2{}_{31}q^1 + \epsilon^2{}_{30}q^0\right),
\end{aligned}\right\} \tag{2.55}$$

which can be rewritten as:

$$\left.\begin{aligned}
\omega^0{}_1 &= -\frac{\kappa}{2}\left(\epsilon_{012}q^2 + \epsilon_{013}q^3\right), \\
\omega^0{}_2 &= -\frac{\kappa}{2}\left(\epsilon_{023}q^3 + \epsilon_{021}q^1\right), \\
\omega^0{}_3 &= -\frac{\kappa}{2}\left(\epsilon_{031}q^1 + \epsilon_{032}q^2\right), \\
\omega^1{}_2 &= \frac{\kappa}{2}\left(\epsilon_{120}q^0 + \epsilon_{123}q^3\right), \\
\omega^1{}_3 &= \frac{\kappa}{2}\left(\epsilon_{132}q^2 + \epsilon_{130}q^0\right), \\
\omega^2{}_3 &= \frac{\kappa}{2}\left(\epsilon_{231}q^1 + \epsilon_{230}q^0\right),
\end{aligned}\right\} \tag{2.56}$$

Therefore we obtain:

$$\left.\begin{aligned}
\omega^0{}_1 &= -\frac{\kappa}{2}\left(q^2 + q^3\right), \\
\omega^0{}_2 &= -\frac{\kappa}{2}\left(q^3 - q^1\right), \\
\omega^0{}_3 &= -\frac{\kappa}{2}\left(-q^1 - q^2\right), \\
\omega^1{}_2 &= \frac{\kappa}{2}\left(q^0 + q^3\right), \\
\omega^1{}_3 &= \frac{\kappa}{2}\left(-q^2 + q^0\right), \\
\omega^2{}_3 &= \frac{\kappa}{2}\left(q^1 + q^0\right)
\end{aligned}\right\} \tag{2.57}$$

the potentials $A^0; A^1; A^2$ and A^3 in regions de ned by Eqs. (2.43) to (2.63). In the Chambers experiment for example, the magnetic eld inside the iron whisker is de ned by Eqs.(2.65) and the potential outside the iron whisker by Eqs.(2.43) to (2.63). Eqs.(2.43) to (2.63) show that in general there is an electric AB e ect and an electromagnetic AB e ect [1] [2]. In regions outside a radar beam for example, there will be a discernible AB e ect to second order in the potential akin to the inverse Faraday e ect [1] [2] in regions where the eld is non-zero. This type of electromagnetic AB e ect is due to the ECE spin eld [1]{ [14] observed in the inverse Faraday e ect. The ECE spin eld is de ned by:

$$F^3 = -gA^1 \wedge A^2 \tag{2.66}$$

and is to SECOND order in the potential. It is a new type of magnetic eld whose origins can now be traced via ECE theory to general relativity. The ECE spin eld does not exist in special relativity (Maxwell Heaviside eld theory), so it shows the general covariance of electromagnetism [1]{ [14]. This means that electromagnetism is a spinning frame of reference which must be described with a non-zero spin connection. The existence of the spin connection is shown experimentally by the well known inverse Faraday e ect (the magnetization of any type of material by a circularly or elliptically polarized electromagnetic eld at any frequency).

In the standard model [16] an attempt is made to explain the magnetic AB e ect using gauge theory in special relativity. In the notation of di erential geometry [1]{ [15] gauge theory means that:

$$A \rightarrow A + dX \tag{2.67}$$

but by the Poincare Lemma:

$$d \wedge dX := 0 \tag{2.68}$$

In the standard model [16] the Stokes Theorem [1]{ [14] [17] is used to claim that:

$$\oint dX = \int_S d \wedge dX \neq 0 \tag{2.69}$$

However, this claim is incorrect [1] [2] because it violates the Stokes Theorem. It is well known that the latter applies [17] to non simply connected spaces, so:

$$\oint dX = \int_S d \wedge dX := 0 \tag{2.70}$$

using the Poincare Lemma (2.68) the Stokes Theorem implies that:

$$\oint dX := 0 \tag{2.71}$$

thus contradicting the claim (2.69) of the standard model.

This error in the standard model can be illustrated for example by the error in Eq.(3.105) of a standard model textbook such as ref. [16]. The task is to evaluate:

$$= \oint \nabla X \cdot dr = \int_S \nabla \times \nabla X := 0 \tag{2.72}$$

As we have seen, Eq.(2.72) must be the correct result by the Stokes Theorem. The function X in the example we are considering here (Eq.3.105 of ref. [16]) is

$$X = \frac{BR^2\theta}{2} \tag{2.73}$$

where B is magnetic flux density, R is a radius and θ is a cylindrical polar coordinate. The cylindrical polar coordinates are related [17] to the Cartesian coordinates by:

$$\left.\begin{array}{cc} \theta = \tan^{-1} y/x, & r = \left(x^2 + y^2\right)^{1/2}, \\ x = r\cos\theta, & y = r\sin\theta \end{array}\right\} \tag{2.74}$$

as is well known. The left hand side of Eq.(2.72) is evaluated between $\theta = 0$ and $\theta = 2\pi$. Therefore:

$$\left.\begin{array}{cc} x = r\cos 2\pi = r, & y = r\sin 2\pi = 0, \\ x = r\cos 0 = r, & y = r\sin 0 = 0 \end{array}\right\} \tag{2.75}$$

and the integral is:

$$\Delta\delta = \oint_r^r \frac{\partial}{\partial x}\left(\tan^{-1}\frac{y}{x}\right)dx + \oint_0^0 \frac{\partial}{\partial y}\left(\tan^{-1}\frac{y}{x}\right)dy = 0 \tag{2.76}$$

This is the correct result as given by the Stokes Theorem (2.72). However, in ref. [16] it is incorrectly asserted that:

$$\oint \boldsymbol{\nabla} X \cdot d\mathbf{r} = ? \qquad [X]_{\theta=0}^{\theta=2\pi} \neq 0 \tag{2.77}$$

The correct way to evaluate Eq.(2.77) is:

$$\oint_{\theta=0}^{\theta=2\pi} \boldsymbol{\nabla} X \cdot d\mathbf{r} = \oint_x^x \boldsymbol{\nabla} X \cdot d\mathbf{r} = 0 \tag{2.78}$$

because

$$x = r\cos\theta, \qquad r = \left(x^2 + y^2\right)^{1/2} \tag{2.79}$$

and:

$$\left.\begin{array}{cc} \text{if} & \theta = 2\pi, \quad r = x, \\ \text{if} & \theta = 0, \quad r = x \end{array}\right\} \tag{2.80}$$

The whole of the argument on the AB effect in ref. [16] is therefore incorrect following the occurrence of this error, both mathematically and physically. The magnetic AB effect is not due to multiply connected spaces in special relativity. It is due to general relativity as we have argued. Thus ECE theory is preferred to the standard model, philosophically, mathematically and experimentally.

The electromagnetic AB effects in ECE theory are summarized as follows: In the Chambers experiment for example the observable is a phase shift:

$$\Delta\phi = \frac{e}{\hbar}\Phi \tag{2.81}$$

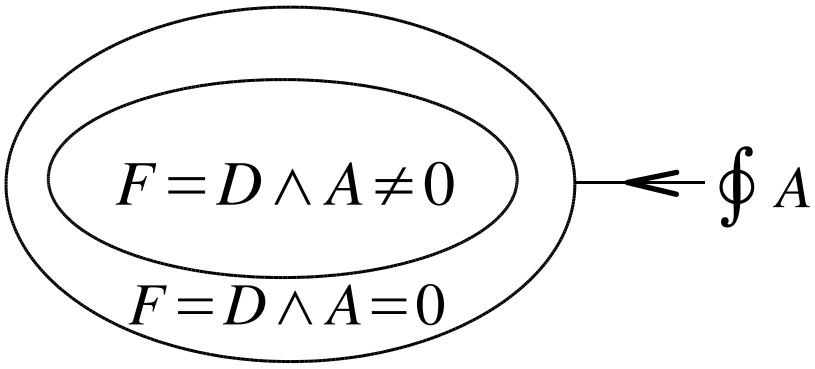

Figure 2.1: AB Effects in ECE Theory

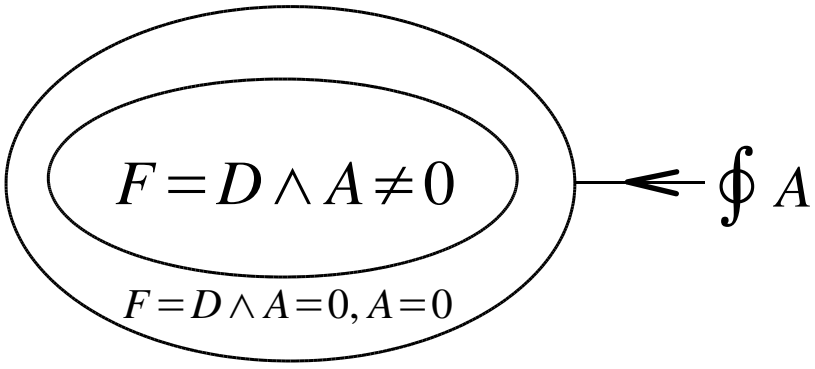

Figure 2.2: Standard Theory

where the magnetic flux is defined by:

$$\Phi = \int_S F = \int_S D \wedge A = \oint A \tag{2.82}$$

The line integral in Eq.(2.82) is around the outer contour of Fig. 2.1, the contour defined by the electron beams of the Chambers experiment [1], [2]. The magnetic flux density is enclosed in the inner area of Fig. 2.1, representing the area of the iron whisker. The latter area is much smaller than the area enclosed by the electron beams. In the area outside the whisker but inside the electron beams there is no magnetic flux density, but there is a potential defined by Eq.(2.63). The AB effect of the Chambers experiment is caused by this potential as we have argued.

In the standard model on the other hand there is no explanation for the Chambers experiment, as summarized in Fig. 2.2 below: As we have seen, the attempted gauge theoretical explanation of the standard model is mathematically incorrect.

2.4 The gravitational AB effect

The gravitational AB effect is defined in ECE theory in regions where:

$$R = D \wedge \omega = 0 \tag{2.83}$$

$$\omega \neq 0 \tag{2.84}$$

Eq.(2.83) may be developed as:

$$d \wedge \omega^{a}{}_{b} + \omega^{a}{}_{c} \wedge \omega^{c}{}_{b} = 0 \tag{2.85}$$

If there is no interaction between gravitation and electromagnetism the Cartan torsion is zero and so the gravitational AB effect in this case is further constrained by:

$$T^{a} = d \wedge q^{a} + \omega^{a}{}_{b} \wedge q^{b} = 0 \tag{2.86}$$

These are the conditions for the EH field theory of centrally directed gravitation. The limit of ECE theory where the Cartan torsion is zero. As is well known for EH theory, the Christoffel connection is symmetric:

$$\Gamma^{\kappa}{}_{\mu\nu} = \Gamma^{\kappa}{}_{\nu\mu} \tag{2.87}$$

and the metric is symmetric:

$$g_{\mu\nu} = g_{\nu\mu} = q^{a}{}_{\mu} q^{b}{}_{\nu} \eta_{ab} \tag{2.88}$$

The symmetry of the spin connection in EH theory is determined by the tetrad postulate [1]– [15]:

$$D_{\mu} q^{a}{}_{\lambda} = \partial_{\mu} q^{a}{}_{\lambda} + \omega^{a}{}_{\mu b} q^{b}{}_{\lambda} - \Gamma^{\nu}{}_{\mu\lambda} q^{a}{}_{\nu} = 0 \tag{2.89}$$

Interchange μ and λ:

$$D_{\lambda} q^{a}{}_{\mu} = \partial_{\lambda} q^{a}{}_{\mu} + \omega^{a}{}_{\lambda b} q^{b}{}_{\mu} - \Gamma^{\nu}{}_{\lambda\mu} q^{a}{}_{\nu} = 0 \tag{2.90}$$

and subtract Eq.(2.90) from Eq.(2.89) to find that:

$$\partial_{\mu} q^{a}{}_{\lambda} + \omega^{a}{}_{\mu b} q^{b}{}_{\lambda} - \left(\partial_{\lambda} q^{a}{}_{\mu} + \omega^{a}{}_{\lambda b} q^{b}{}_{\mu} \right) = 0 \tag{2.91}$$

This result is consistent with the fact that there is no Cartan torsion in EH theory:

$$T^{a}{}_{\mu\lambda} = \partial_{\mu} q^{a}{}_{\lambda} + \omega^{a}{}_{\mu b} q^{b}{}_{\lambda} = 0 \tag{2.92}$$

$$T^{a}{}_{\lambda\mu} = \partial_{\lambda} q^{a}{}_{\mu} + \omega^{a}{}_{\lambda b} q^{b}{}_{\mu} = 0 \tag{2.93}$$

Eqs.(2.92) to (2.93) must be symmetric under interchange of μ and λ, so

$$\omega^{a}{}_{\mu b} q^{b}{}_{\lambda} = \omega^{a}{}_{\lambda b} q^{b}{}_{\mu} \tag{2.94}$$

i.e.

$$\omega^{a}{}_{\mu\lambda} = \omega^{a}{}_{\lambda\mu} \tag{2.95}$$

Now interchange the indices a and b to find that:

$$\omega^{b}{}_{\mu\lambda} = \omega^{b}{}_{\lambda\mu} \tag{2.96}$$

Eqs.(2.95) and (2.96) are the same, so the spin connection in EH theory must be symmetric:

$$\omega^{a}{}_{\mu b} = \omega^{b}{}_{\mu a} \tag{2.97}$$

Therefore the gravitational AB equations for the diagonal elements of the spin connection are:

$$\left.\begin{array}{l} d \wedge \omega^{0}{}_{0} + \omega^{0}{}_{c} \wedge \omega^{c}{}_{0} = 0, \\ d \wedge \omega^{1}{}_{1} + \omega^{1}{}_{c} \wedge \omega^{c}{}_{1} = 0, \\ d \wedge \omega^{2}{}_{2} + \omega^{2}{}_{c} \wedge \omega^{c}{}_{2} = 0, \\ d \wedge \omega^{3}{}_{3} + \omega^{3}{}_{c} \wedge \omega^{c}{}_{3} = 0 \end{array}\right\} \tag{2.98}$$

and there is a similar set of equations for the off diagonal elements. The gravitational AB effect has been observed experimentally [1] [2].

Acknowledgements The British Parliament and Head of State, Queen Elizabeth II, are thanked for a Civil List pension. The voluntary staff of AIAS are thanked for many interesting discussions and Franklin Amador for accurate and voluntary typesetting.

Bibliography

[1] M. W. Evans, Generally Covariant Unified Field Theory (Abramis, 2005), vol. 1, ibid., vols. 2 and 3 (Abramis, 2006), vol. 4.

[2] L. Felker, The ECE Equations of Unified Field Theory (preprint on www.aias.us and www.atomicprecision.com). H. Eckardt and L. Felker, popular article on these sites.

[3] M. W. Evans, Found. Phys. Lett., 16, 367, 507 (2003).

[4] M. W. Evans, Found. Phys. Lett., 17, 25, 149, 267, 301, 393, 433, 535, 663 (2004).

[5] M. W. Evans, Found. Phys. Lett., 18, 139, 259, 519 (2005).

[6] M. W. Evans and L. B. Crowell, Classical and Quantum Electrodynamics and the $\boldsymbol{B}^{(3)}$ Field, (World Scientific, Singapore, 2001).

[7] M. W. Evans (ed.), Modern Non-Linear Optics, a special topical issue in three parts of I. Prigogine and S. A. Rice (series eds.), Advances in Chemical Physics (Wiley Interscience, New York, 2001, 2nd ed., hardback and e book), vols. 119(1) - 119(3).

[8] M. W. Evans and J.-P. Vigier, The Enigmatic Photon, (Kluwer, Dordrecht, 1994 to 2002, hardback and softback), in five volumes.

[9] M. W. Evans and A. A. Hasanein, The Photomagneton in Quantum Field Theory (World Scientific, 1994).

[10] M. W. Evans and S. Kielich (eds.), first edition of ref. (7) (Wiley Interscience, New York, 1992, reprinted 1993, softback 1997), vols. 85(1) - 85(3).

[11] M. W. Evans, Physica B, 182, 227, 237 (first papers on the ECE spin field).

[12] M. W. Evans, Generally Covariant Dynamics, (preprint on www.aias.us and www.atomicprecision.com).

[13] M. W. Evans et al., preprints of seventy-two papers on ECE theory (www.aias.us and www.atomicprecision.com).

[14] Feedback sites for www.aias.us and www.atomicprecision.com, indicating about two million hits a year.

[15] S. P. Carroll, "Space-time and Geometry: an Introduction to General Relativity" (Addison Wesley, New York, 2004).

[16] . L. H Ryder, Quantum Field Theory, (Cambridge, 1996, 2nd ed.).

[17] E. G. Milewski (ed.), The Vector Analysis Problem Solver (REA, New York City, 1987), for example problem 17-27.