THE BELTRAMI STRUCTURE OF ELECTROMAGNETISM AND GRAVITATION.

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ABSTRACT

It is shown that in electromagnetism in ECE physics the magnetic flux density, vector potential and spin connection are in general Beltrami vectors. The vector part of the Cartan identity also has a Beltrami structure in general. In U(1) physics (standard model), the Beltrami structure of the magnetic flux densit y immediately refutes U(1) gauge invariance and indicates identically non-zero photon mass and the absence of a Higgs boson. In magnetostatics the spin curvature and current densit yare also Beltrami vectors. Photon mass theory is discussed and developed in terms of Beltrami electrodynamics and the Proca equation. In general, the Beltrami equation has intricate solutions which are discussed, graphed and animated in Section 3. Similar conclusions hold for gravitation

Keywords: ECE physics, Beltrami electrodynamics, finite photon mass.

UFT 258

1. INTRODUCTION.

In recent papers of this series the Beltrami structure of ECE physics has been investigated, with several interesting conclusions {1 - 10}, In this paper it is shown that in electromagnetism in general, the magnetic flux density, vector potential and spin connection vector are always Beltrami vectors with intricate structures in general, solutions of the Beltrami equation {11}. The latter was first applied in hydrodynamics, and is useful in several subject areas that include magnetohydrodynamics, aerodynamics, cosmology and as shown in recent papers {1 - 10}, electromagnetism and gravitation. The background notes for this paper (UFT258 on www.aias.us) contain many details and conclusions and are referred to in context. In Section 2 the Beltrami structure of the vector potential and spin connection vector is proven in ECE physics from the Beltrami structure of the magnetic flux density $\underline{B}$. It is shown that the space part of the Cartan identity also has a Beltrami structure. In U(1) physics (the standard model), the Beltrami structure of $\underline{B}$ immediately refutes U(1) gauge invariance because $\underline{B}$ becomes directly proportional to $\underline{A}$. It follows that the photon mass is identically non zero, however small in magnitude. Therefore there is no Higgs boson in nature because the latter is the result of U(1) gauge invariance. The Beltrami structure of $\underline{B}$ is the direct result of the Gauss law of magnetism and the absence of a magnetic monopole. It is difficult to conceive of why U(1) gauge invariance should ever have been adopted as a theory, because its refutation is trivial. Once U(1) gauge invariance is discarded, a rich panoply of new ideas and results emerge. These are summarized briefly in Section 2 with reference to the complete calculational details in the background notes accompanying UFT258 on www.aias.us.

In Section 3 the intricate structure of Beltrami vectors is illustrated by graphics and animation. The animations are all posted in the publication / animation section of www.aias.us. This paper shows that this intricate structure is present in electromagnetism and gravitation, in the magnetic flux density, vector potential, and spin connection vector. These are major discoveries of ECE physics.

2. DERIVATION OF THE BELTRAMI EQUATIONS.

The Gauss law of magnetism in ECE physics is:

$$\underline{\nabla} \cdot \underline{B}^a = 0 \qquad -(1)$$

from which the Beltrami equation follows immediately (notes 258(1) to 258(5), and 258(9)):

$$\underline{\nabla} \times \underline{B}^a = \kappa \underline{B}^a . \qquad -(2)$$

Here $\underline{B}^a$ is the magnetic flux density in tesla and κ has the units of inverse metres. In the simplest case κ is a wave vector, but as the accompanying notes 285(6) to 258(8) show it can become very intricate. Some well known solutions {11} of the Eq. (2) are animated in Section 3, so the flow structures can be viewed directly. Comparing Eq. (2) with the Ampère Maxwell law of ECE physics:

$$\underline{\nabla} \times \underline{B}^a - \frac{1}{c^2} \frac{\partial \underline{E}^a}{\partial t} = \mu_0 \underline{J}^a \qquad -(3)$$

the magnetic flux density is given directly as follows:

$$\underline{B}^a = \frac{1}{\kappa} \left(\frac{1}{c^2} \frac{\partial \underline{E}^a}{\partial t} + \mu_0 \underline{J}^a \right) \qquad -(4)$$

where $\underline{E}^a$ is the electric field strength in volts per meter and $\underline{J}^a$ is the current density. Here μ_0 is the vacuum permeability in S. I. Units. The Coulomb law in ECE physics is:

$$\underline{\nabla} \cdot \underline{E}^a = \rho^a / \epsilon_0 \qquad -(5)$$

where ρ^a is the charge density and ϵ_0 the vacuum permittivity. Using Eq. (5):

$$\underline{\nabla} \cdot \underline{B} = \frac{\mu_0}{k} \left(\frac{\partial \rho^a}{\partial t} + \underline{\nabla} \cdot \underline{J}^a \right) = 0 \quad -(6)$$

a result which follows from:

$$\epsilon_0 \mu_0 = \frac{1}{c^2} \quad -(7)$$

where c is the universal constant known as the vacuum speed of light. Note carefully that c is the vacuum speed of light if and only if the photon mass is identically zero. The latter assumption relies on U(1) gauge invariance. The conservation of charge current density in ECE physics is:

$$\frac{\partial \rho^a}{\partial t} + \underline{\nabla} \cdot \underline{J}^a = 0 \quad -(8)$$

so $\underline{B}^a$ is always a Beltrami vector, Q. E. D.

In U(1) physics:

$$\underline{B} = \underline{\nabla} \times \underline{A}, \quad -(9)$$

$$\underline{\nabla} \times \underline{B} = k \, \underline{\nabla} \times \underline{A} \quad -(10)$$

where $\underline{A}$ is the vector potential. Eqs. (9) and (10) immediately show that in U(1) physics, the vector potential also obeys a Beltrami equation:

$$\underline{\nabla} \times \underline{A} = k \underline{A} \quad -(11)$$

and

$$\underline{B} = k \underline{A}. \quad -(12)$$

So the magnetic flux density in U(1) physics is directly proportional to the vector potential $\underline{A}$. It follows immediately (note 258(1)) that $\underline{A}$ cannot be U(1) gauge invariant, because U(1) gauge invariance means:

$$\underline{A} \rightarrow \underline{A} + \underline{\nabla} \psi \quad -(13)$$

and if $\underline{A}$ is changed, $\underline{B}$ is changed. The obsolete dogma of U(1) physics asserted that Eq. (13) does not change any physical quantity. This dogma is obviously incorrect because $\underline{B}$ is a physical quantity and Eq. (13) changes it. Therefore there is finite photon mass and no

Higgs boson. The expenditure of tens of billions at CERN is trivially refuted.

Finite photon mass theory relies on the Proca equation {1 -10}, which is not U(1) gauge invariant {12}. The Proca equation can be developed as:

$$\underline{\nabla} \cdot \underline{B}^a = 0 \quad -(14)$$

$$\underline{\nabla} \times \underline{E}^a + \partial \underline{B}^a / \partial t = \underline{0} \quad -(15)$$

$$\underline{\nabla} \cdot \underline{E}^a = \rho^a / \epsilon_0 \quad -(16)$$

$$\underline{\nabla} \times \underline{B}^a - \frac{1}{c^2} \frac{\partial \underline{E}^a}{\partial t} = \mu_0 \underline{J}^a \quad -(17)$$

where the four current density is:

$$J^{a\mu} = (c\rho^a, \underline{J}^a) \quad -(18)$$

and where the four potential is:

$$A^{a\mu} = \left(\frac{\phi^a}{c}, \underline{A}^a \right). \quad -(19)$$

Proca theory in ECE physics asserts that {1 -10}:

$$J^{a\mu} = -\epsilon_0 \left(\frac{mc}{\hbar} \right)^2 A^{a\mu} \quad -(20)$$

where m is the photon mass and $\hbar$ the reduced Planck constant. Therefore:

$$\rho^a = -\epsilon_0 c^2 (mc/\hbar)^2 \phi^a \quad -(21)$$

$$\underline{J}^a = -\epsilon_0 (mc/\hbar)^2 \underline{A}^a \quad -(22)$$

The Proca equation was inferred in the mid thirties but is almost entirely absent from the textbooks. This is a most unfortunate result of incorrect dogma, that the photon mass is zero, despite being postulated by Einstein in about 1905 to be a particle. Therefore it is convenient to give a brief review of Proca theory on the U(1) level before proceeding to the ECE level. The U(1) Proca field equation in correct S. I. Units is

$$\partial_\mu F^{\mu\nu} = \frac{J^\nu}{\epsilon_0} = -\left(\frac{mc}{\hbar}\right)^2 A^\nu \quad -(23)$$

It follows immediately that:

$$\partial_\nu \partial_\mu F^{\mu\nu} = \frac{1}{\epsilon_0}\partial_\nu J^\nu = -\left(\frac{mc}{\hbar}\right)^2 \partial_\nu A^\nu = 0 \quad -(24)$$

and that:

$$\partial_\mu J^\mu = \partial_\mu A^\mu = 0. \quad -(25)$$

Eq. (25a) is conservation of charge current density and Eq. (25b) is the Lorenz condition. In the Proca equation, the Lorenz condition has nothing to do with gauge invariance. The U(1) gauge invariance means that:

$$A^\mu \rightarrow A^\mu + \partial^\mu \chi \quad -(26)$$

and from Eq. (23) it is trivially apparent that the Proca field and charge current density change under the transformation (26), so are not gauge invariant, Q. E. D. The entire edifice of U(1) electrodynamics collapses as soon as photon mass is considered.

In vector notation Eq. (24) is:

$$\frac{1}{c}\frac{\partial}{\partial t}\left(\underline{\nabla}\cdot\underline{E}\right) = \frac{1}{c\epsilon_0}\frac{\partial \rho}{\partial t} = 0 \quad -(27)$$

and:

$$\underline{\nabla}\cdot\underline{\nabla}\times\underline{B} - \frac{1}{c^2}\frac{\partial}{\partial t}\left(\underline{\nabla}\cdot\underline{E}\right) = \mu_0 \underline{\nabla}\cdot\underline{J} = 0 \quad -(28)$$

Now use

$$\underline{\nabla}\cdot\underline{\nabla}\times\underline{B} = 0 \quad -(29)$$

and the U(1) Coulomb law:

$$\underline{\nabla} \cdot \underline{E} = \rho / \epsilon_0 \quad -(30)$$

to find that

$$-\frac{1}{c^2 \epsilon_0} \frac{\partial \rho}{\partial t} = \mu_0 \underline{\nabla} \cdot \underline{J} \quad -(31)$$

which is the equation of charge current conservation, Q. E. D.:

$$\frac{\partial \rho}{\partial t} + \underline{\nabla} \cdot \underline{J} = 0. \quad -(32)$$

In the Proca theory, Eq. (25) implies the Lorenz gauge, as it is known in standard physics:

$$\partial_\mu A^\mu = \frac{1}{c^2} \frac{\partial \phi}{\partial t} + \underline{\nabla} \cdot \underline{A} = 0. \quad -(33)$$

The Proca wave equation in U(1) or standard physics is obtained from the U(1) definition of the field tensor:

$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu. \quad -(34)$$

So:

$$\partial_\mu (\partial^\mu A^\nu - \partial^\nu A^\mu) = \square A^\nu - \partial^\nu \partial_\mu A^\mu = -(mc/\hbar)^2 A^\nu \quad -(35)$$

in which:

$$\partial_\mu A^\mu = 0 \quad -(36)$$

Eq. (36) follows from Eq. (23) in Proca physics, but in standard physics the Lorenz gauge has to be assumed, and is arbitrary. So the Proca wave equation is:

$$(\square + (mc/\hbar)^2) A^\nu = 0. \quad -(37)$$

In ECE physics Eq. (37) is derived from the tetrad postulate of Cartan geometry and becomes:

$$\left(\square + \left(\frac{mc}{\hbar}\right)^2 \right) A^a_\mu = 0. \quad -(38)$$

In ECE physics the conservation of charge current density is:

$$\partial_\mu J^{a\mu} = 0 \quad -(39)$$

and is consistent with Eqs. (16) and (17) as just shown on the U(1) level, Q. E. D.

ECE physics has been tested rigorously in two hundred and fifty eight papers of this series to date, and is always self consistent because it is based on Cartan's self consistent geometry.

In ECE physics the electric charge density is geometrical in origin and is:

$$\rho^a = \epsilon_0 \left(\underline{\omega}^a{}_b \cdot \underline{E}^b - c\underline{A}^b \cdot \underline{R}^a{}_b(\mathrm{orb}) \right) \quad -(40)$$

and the electric current density is:

$$\underline{J}^a = \frac{1}{\mu_0} \left(\underline{\omega}^a{}_b \times \underline{B}^b + \frac{\omega_0}{c}\underline{E}^b - \underline{A}^b \times \underline{R}^a{}_b(\mathrm{spin}) - A_0^b \underline{R}^a{}_b(\mathrm{orb}) \right) \quad -(41)$$

Here $R^a{}_b(\mathrm{spin})$ and $R^a{}_b(\mathrm{orb})$ are the spin and orbital components of the curvature tensor {1 -10}. So Eqs. (8), (40) and (41) give many new equations of ECE physics which can be developed systematically in future work.

In magnetostatics for example the relevant equations are:

$$\underline{\nabla} \cdot \underline{B}^a = 0 \quad -(42)$$

$$\underline{\nabla} \times \underline{B}^a = \mu_0 \underline{J}^a \quad -(43)$$

and:

$$\underline{\nabla} \cdot \underline{J}^a = \underline{\nabla} \cdot \underline{\nabla} \times \underline{B}^a = 0 \quad -(44)$$

So it follows from charge current conservation that:

$$\partial \rho^a / \partial t = 0 . \quad -(45)$$

If it is assumed that the scalar potential is zero in magnetostatics, the usual assumption, then

$$\underline{J}^a = \frac{1}{\mu_0}\left(\underline{\omega}^a{}_b \times \underline{B}^b - \underline{A}^b \times \underline{R}^a{}_b(\text{spin})\right) - (46)$$

because there is no electric field present. It follows from Eqs (44) and (46) that:

$$\underline{\nabla} \cdot \underline{\omega}^a{}_b \times \underline{B}^b = \underline{\nabla} \cdot \underline{A}^b \times \underline{R}^a{}_b(\text{spin}) - (47)$$

in ECE magnetostatics.

In immediately preceding papers of this series it has been shown that in the absence of a magnetic monopole:

$$\underline{\omega}^a{}_b \cdot \underline{B}^b = \underline{A}^b \cdot \underline{R}^a{}_b(\text{spin}) - (48)$$

and that the space part of the Cartan identity in the absence of a magnetic monopole gives the two equations:

$$\underline{\nabla} \cdot \underline{\omega}^a{}_b \times \underline{A}^b = 0 \quad -(49)$$

and:

$$\underline{\omega}^a{}_b \cdot \underline{\nabla} \times \underline{A}^b = \underline{A}^b \cdot \underline{\nabla} \times \underline{\omega}^a{}_b . \quad -(50)$$

In ECE physics the magnetic flux density is:

$$\underline{B}^a = \underline{\nabla} \times \underline{A}^a - \underline{\omega}^a{}_b \times \underline{A}^b - (51)$$

so the Beltrami equation gives:

$$\underline{\nabla} \times \underline{B}^a = \kappa \underline{B}^a = \kappa\left(\underline{\nabla} \times \underline{A}^a - \underline{\omega}^a{}_b \times \underline{A}^b\right) - (52)$$

Eq. (49) from the space part of the Cartan identity is also a Beltrami equation, as is any divergenceless equation:

$$\underline{\nabla} \times \left(\underline{\omega}^a{}_b \times \underline{A}^b \right) = \kappa\, \underline{\omega}^a{}_b \times \underline{A}^b \quad -(53)$$

From Eq. (52):

$$\underline{\nabla} \times \left(\underline{\nabla} \times \underline{A}^a \right) - \underline{\nabla} \times \left(\underline{\omega}^a{}_b \times \underline{A}^b \right) = \kappa \left(\underline{\nabla} \times \underline{A}^a - \underline{\omega}^a{}_b \times \underline{A}^b \right) \quad -(54)$$

Using Eq. (53):

$$\underline{\nabla} \times \left(\underline{\nabla} \times \underline{A}^a \right) = \kappa\, \underline{\nabla} \times \underline{A}^a \quad -(55)$$

which implies that the vector potential is also defined in general by a Beltrami equation:

$$\underline{\nabla} \times \underline{A}^a = \kappa\, \underline{A}^a \quad -(56)$$

Q. E. D. This is a generally valid result of ECE physics which implies that:

$$\underline{\nabla} \cdot \underline{A}^a = 0 \quad -(57)$$

From Eq. (25), it follows that:

$$\frac{\partial \rho^a}{\partial t} = 0 \quad -(58)$$

is a general result of ECE physics.

From Eqs. (50) and (56):

$$\underline{\nabla} \times \underline{\omega}^a{}_b = \kappa\, \underline{\omega}^a{}_b \quad -(59)$$

so the spin connection vector of ECE physics is also defined in general by a Beltrami equation.

These important results can be cross checked for internal consistency using note 258(4), starting from Eq. (50) of this paper. Considering the X component for example:

$$\omega^a{}_{Xb}\left(\underline{\nabla}\times\underline{A}^b\right)_X = A^b_X\left(\underline{\nabla}\times\underline{\omega}^a{}_b\right)_X \quad -(60)$$

and it follows that:

$$\frac{1}{A^{(1)}_X}\left(\underline{\nabla}\times\underline{A}^{(1)}\right)_X = \frac{1}{\omega^a{}_{X(1)}}\left(\underline{\nabla}\times\underline{\omega}^a{}_{(1)}\right)_X \quad -(61)$$

and similarly for the Y and Z components. In order for this to be a Beltrami equation, Eqs. (56) and (59) must be true, Q. E. D.

In magnetostatics there are additional results which emerge as follows. From vector analysis:

$$\underline{\nabla}\cdot\underline{\omega}^a{}_b\times\underline{B}^b = \underline{B}^b\cdot\underline{\nabla}\times\underline{\omega}^a{}_b - \underline{\omega}^a{}_b\cdot\underline{\nabla}\times\underline{B}^b \quad -(62)$$

and

$$\underline{\nabla}\cdot\underline{A}^b\times\underline{R}^a{}_b(\text{spin}) = \underline{R}^a{}_b(\text{spin})\cdot\underline{\nabla}\times\underline{A}^a - \underline{A}^b\cdot\underline{\nabla}\times\underline{R}^a{}_b(\text{spin}). \quad -(63)$$

It is immediately clear that Eqs. (2) and (59) give Eq. (62) self consistently, Q. E. D. Eq. (63) gives:

$$\underline{\nabla}\cdot\underline{\omega}^a{}_b\times\underline{B}^b = \underline{\nabla}\cdot\underline{A}^b\times\underline{R}^a{}_b(\text{spin}) = 0 \quad -(64)$$

and using Eq. (63):

$$\underline{\nabla}\times\underline{R}^a{}_b(\text{spin}) = \kappa\,\underline{R}^a{}_b(\text{spin}) \quad -(65)$$

so the spin curvature is defined by a Beltrami equation in magnetostatics. Also in magnetostatics:

$$\underline{\nabla}\times\underline{B}^a = \kappa\,\underline{B}^a = \mu_0\,\underline{J}^a \quad -(66)$$

so it follows that the current density of magnetostatics is also defined by a Beltrami equation:

$$\underline{\nabla} \times \underline{J}^a = \kappa \underline{J}^a \quad -(67)$$

All these Beltrami equations have in general the intricate flow structures animated in Section 3 and posted in publications / animation on www.aias.us. As discussed in Eqs. (31) to (35) of Note 258(5) plane wave structures and O(3) electrodynamics {1 - 10} are also defined by Beltrami equations. The latter give simple solutions for vacuum plane waves. In other cases the solutions become intricate. The B(3) field {1 - 10} is defined by the simplest type of Beltrami equation:

$$\underline{\nabla} \times \underline{B}^{(3)} = 0 \underline{B}^{(3)} \quad -(68)$$

In photon mass theory therefore:

$$\underline{\nabla} \times \underline{A}^a = \kappa \underline{A}^a \quad -(69)$$

and:

$$\left(\Box + \left(\frac{mc}{\hbar} \right)^2 \right) \underline{A}^a = \underline{0} \quad -(70)$$

It follows from Eq. (69) that:

$$\underline{\nabla} \cdot \underline{A}^a = 0 \quad -(71)$$

so:

$$\underline{\nabla} \times (\underline{\nabla} \times \underline{A}^a) = \kappa \underline{\nabla} \times \underline{A}^a = \kappa^2 \underline{A}^a \quad -(72)$$

produces the Helmholtz wave equation {11}:

$$\nabla^2 \underline{A}^a + \kappa^2 \underline{A}^a = \underline{0}. \quad -(73)$$

Eq. (70) is:

$$\left(\frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \nabla^2 + \left(\frac{mc}{\hbar} \right)^2 \right) \underline{A}^a = \underline{0} \quad -(74)$$

so:

$$\left(\frac{1}{c^2} \frac{\partial^2}{\partial t^2} + \kappa^2 + \left(\frac{mc}{\hbar} \right)^2 \right) \underline{A}^a = \underline{0}. \quad -(75)$$

Now use:

$$\underline{p} = \hbar \underline{\kappa} \quad -(76)$$

and

$$\frac{\partial^2}{\partial t^2} = -\frac{E^2}{\hbar^2} \quad -(77)$$

to find that Eq. (75) is the Einstein energy equation for the photon of mass m, so the analysis is rigorously self consistent, Q. E. D.

In ECE physics the Lorenz gauge is:

$$\partial_\mu A^{a\mu} = 0 \quad -(78)$$

i. e.

$$\frac{1}{c^2} \frac{\partial \phi^a}{\partial t} + \underline{\nabla} \cdot \underline{A}^a = 0 \quad -(79)$$

with the solution:

$$\frac{\partial \phi^a}{\partial t} = \underline{\nabla} \cdot \underline{A}^a = 0. \quad -(80)$$

This is again a general result of ECE physics applicable under any circumstances. Also in

ECE physics in general the spin connection vector has no divergence:

$$\underline{\nabla} \cdot \underline{\omega}^a{}_b = 0 \quad -(81)$$

because:

$$\underline{\nabla} \times \underline{\omega}^a{}_b = \kappa\, \underline{\omega}^a{}_b . \quad -(82)$$

Another rigorous test for self consistency is given by the definition of the magnetic field in ECE physics:

$$\underline{B}^a = \underline{\nabla} \times \underline{A}^a - \underline{\omega}^a{}_b \times \underline{A}^a \quad -(83)$$

so:

$$\underline{\nabla} \cdot \underline{B}^a = -\underline{\nabla} \cdot \underline{\omega}^a{}_b \times \underline{A}^b = 0 . \quad -(84)$$

By vector analysis:

$$\begin{aligned}\underline{\nabla} \cdot \underline{\omega}^a{}_b \times \underline{A}^a &= \underline{A}^b \cdot \underline{\nabla} \times \underline{\omega}^a{}_b - \underline{\omega}^a{}_b \cdot \underline{\nabla} \times \underline{A}^b \\ &= 0 \end{aligned} \quad -(85)$$

because:

$$\underline{\nabla} \times \underline{\omega}^a{}_b = \kappa\, \underline{\omega}^a{}_b , \quad -(86)$$

$$\underline{\nabla} \times \underline{A}^b = \kappa\, \underline{A}^b , \quad -(87)$$

and:

$$\underline{\nabla} \cdot \underline{A}^b = 0 , \quad -(88)$$

$$\underline{\nabla} \cdot \underline{\omega}^a{}_b = 0 . \quad -(89)$$

In the absence of a magnetic monopole Eq. (84) also follows from the space part of the Cartan identity. So the entire analysis is rigorously self consistent.

The cross consistency of the Beltrami and ECE equations can be checked using:

$$\underline{B}^b = \kappa \underline{A}^b - \underline{\omega}^b{}_c \times \underline{A}^c \quad -(90)$$

(as in note 258(1)). Eq. (90) follows from Eqs. (83) and (87). Multiply Eq. (90) by $\underline{\omega}^a{}_b$ and use Eq. (48) to find:

$$\kappa \underline{\omega}^a{}_b \cdot \underline{A}^b - \underline{\omega}^a{}_b \cdot \underline{\omega}^b{}_c \times \underline{A}^c = \underline{A}^b \cdot \underline{R}^a{}_b(\text{spin}). \quad -(91)$$

Now use:

$$\underline{\omega}^a{}_b \cdot \underline{\omega}^b{}_c \times \underline{A}^c = \underline{A}^c \cdot (\underline{\omega}^a{}_b \times \underline{\omega}^b{}_c) \quad -(92)$$

and relabel summation indices to find:

$$\kappa \underline{\omega}^a{}_b \cdot \underline{A}^b - \underline{A}^b \cdot (\underline{\omega}^a{}_c \times \underline{\omega}^c{}_b) = \underline{A}^b \cdot \underline{R}^a{}_b(\text{spin}). \quad -(93)$$

It follows that:

$$\underline{R}^a{}_b(\text{spin}) = \kappa \underline{\omega}^a{}_b - \underline{\omega}^a{}_c \times \underline{\omega}^c{}_b = \underline{\nabla} \times \underline{\omega}^a{}_b - \underline{\omega}^a{}_c \times \underline{\omega}^c{}_b \quad -(94)$$

Q. E. D. The analysis correctly and self consistently produces the correct definition of the spin curvature.

Finally, on the U(1) level for the sake of illustration, consider the Beltrami equations (note 258(3)):

$$\underline{\nabla} \times \underline{A} = \kappa \underline{A} \quad -(95)$$

and:

$$\underline{\nabla} \times \underline{B} = \kappa \underline{B} \quad -(96)$$

In the Ampère Maxwell law:

$$\underline{\nabla} \times \underline{B} - \frac{1}{c^2}\frac{\partial \underline{E}}{\partial t} = \mu_0 \underline{J} \quad -(97)$$

It follows that:

$$\kappa^2 \underline{A} = \underline{J} + \frac{1}{c^2}\frac{\partial \underline{E}}{\partial t} \quad -(98)$$

where:

$$\underline{E} = -\underline{\nabla}\phi - \frac{\partial \underline{A}}{\partial t}. \quad -(99)$$

Therefore:

$$\kappa^2 \underline{A} = \mu_0 \underline{J} + \frac{1}{c^2}\frac{\partial}{\partial t}\left(-\underline{\nabla}\phi - \frac{\partial \underline{A}}{\partial t}\right) \quad -(100)$$

and using the Lorenz condition:

$$\underline{\nabla} \cdot \underline{A} + \frac{1}{c^2}\frac{\partial \phi}{\partial t} = 0 \quad -(101)$$

it follows that:

$$\partial \phi / \partial t = 0. \quad -(102)$$

Using:

$$\Box = \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2, \quad -(103)$$

Eq. (100) becomes the d'Alembert equation in the presence of current density:

$$\Box \underline{A} = \mu_0 \underline{J} \quad -(104)$$

The solutions of the d'Alembert equation (104) may be found from:

$$\underline{B} = \kappa \underline{A} \quad -(105)$$

showing in another way that as soon as the Beltrami equation (2) is used, U(1) gauge invariance is refuted.

3. GRAPHICS AND ANIMATION

Section by co author Horst Eckardt

The Beltrami structure of electromagnetism and gravitation

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3 Graphics and animation

1. We start the investigation of special Beltrami fields with a general consideration. Marsh[12] defines a general Beltrami field with cylindrical geometry by

$$\mathbf{B} = \begin{bmatrix} 0 \\ B_\theta\,(r) \\ B_Z\,(r) \end{bmatrix} \tag{106}$$

with cylindrical coordinates r, θ, Z. There is only an r dependence of the field components. For this to be a Beltrami field, the Beltrami condition in cylindrical coordinates

$$\boldsymbol{\nabla} \times \mathbf{B} = \begin{bmatrix} \frac{1}{r}\frac{\partial B_Z}{\partial \theta} - \frac{\partial B_\theta}{\partial Z} \\ \frac{\partial B_r}{\partial Z} - \frac{\partial B_Z}{\partial r} \\ \frac{1}{r}\left(\frac{\partial (r\,B_\theta)}{\partial r} - \frac{\partial B_r}{\partial \theta}\right) \end{bmatrix} = \kappa\,\mathbf{B} \tag{107}$$

must hold. The divergence in cylindrical coordinates is

$$\boldsymbol{\nabla} \cdot \mathbf{B} = \frac{1}{r}\frac{\partial (r\,B_r)}{\partial r} + \frac{1}{r}\frac{\partial B_\theta}{\partial \theta} + \frac{\partial B_Z}{\partial Z}. \tag{108}$$

Obviously the field (106) is divergence-free, a prerequisite to be a Beltrami field. Eq.(107) simplifies to

$$\boldsymbol{\nabla} \times \mathbf{B} = \begin{bmatrix} 0 \\ -\frac{\partial B_Z}{\partial r} \\ \frac{\partial B_\theta}{\partial r} + \frac{1}{r} B_\theta . \end{bmatrix} = \kappa \begin{bmatrix} 0 \\ B_\theta \\ B_Z \end{bmatrix}. \tag{109}$$

Since the common factor κ (in general a function) must be identical for the second and third component, we obtain the condition

$$-\frac{\frac{\partial}{\partial r}\,B_Z}{B_\theta} = \frac{r\,(\frac{\partial}{\partial r}\,B_\theta) + B_\theta}{r\,B_Z} \tag{110}$$

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or

$$-r\,B_Z\,\left(\frac{\partial}{\partial\,r}\,B_Z\right)=B_\theta\,\left(r\,\left(\frac{\partial}{\partial\,r}\,B_\theta\right)+B_\theta\right) \tag{111}$$

which can be transformed into the integral equation

$$-\frac{{B_Z}^2}{2}=\int\left(B_\theta\,\left(\frac{\partial}{\partial\,r}\,B_\theta\right)+\frac{{B_\theta}^2}{r}\right)\,dr+C. \tag{112}$$

From this non-linear differential equation one of the B components can be obtained when the other is known. We tried out several calculations by computer algebra. It is not easy to find meaningful physical solutions. A very simple case is

$$B_\theta=\alpha r \tag{113}$$

with a constant α. Then from Eq.(112) follows (with $C=0$):

$$-\frac{{B_Z}^2}{2}=2\;\alpha^2\;\int r\;dr=\alpha^2\,r^2 \tag{114}$$

or

$$B_Z=\pm\sqrt{2}\,i\,\alpha\,r \tag{115}$$

which is a complex solution with a complex wave number function

$$\kappa=\mp\frac{\sqrt{2}\,i}{r}. \tag{116}$$

Using other powers of r results in complex solutions also. Trigonometric functions lead to very complicated solutions of the integral - if any - and are barely manageable even by computer algebra.

Next we consider the case of constant κ. From the second component of Eq.(109) follows

$$-\frac{\partial}{\partial\,r}\,B_Z=\kappa\,B_\theta \tag{117}$$

and from the third component

$$r\,\frac{\partial}{\partial\,r}\,B_\theta+B_\theta=\kappa\,r\,B_Z. \tag{118}$$

Integrating Eq.(117), inserting the result for B_Z into (118) gives

$$\frac{\partial}{\partial\,r}\,B_\theta+\frac{B_\theta}{r}=-\kappa^2\,\int B_\theta\;dr, \tag{119}$$

and differentiating this equation leads to the second order differential equation

$$r^2\,\frac{\partial^2}{\partial\,r^2}\,B_\theta+r\,\frac{\partial}{\partial\,r}\,B_\theta+\kappa^2\,r^2\,B_\theta-B_\theta=0. \tag{120}$$

Finally we change the variable r to κr which leads to Bessel's differential equation

$$r^2 \frac{d^2}{d\,r^2} B_\theta(\kappa\,r) + r\,\frac{d}{d\,r} B_\theta(\kappa\,r) + \left(\kappa^2\,r^2 - 1\right)\,B_\theta(\kappa\,r) = 0. \tag{121}$$

The solution is the Bessel function

$$B_\theta(r) = B_0\,J_1(\kappa r) \tag{122}$$

(with a constant B_0) and from (117) follows

$$B_Z(r) = B_0\,J_0(\kappa r). \tag{123}$$

This is the known solution of Reed/Marsh, scaled by the wave number κ, with longitudinal components. This solution was already analyzed in paper 257. The stream lines are shown in Fig. 1. It has to be taken in mind that stream lines show how a test particle moves in the vector field which is considered a velocity field:

$$\mathbf{x} + \Delta\mathbf{x} = \mathbf{x} + \mathbf{v}(\mathbf{x})\,\Delta t. \tag{124}$$

All streamline examples are started with 9 points in parallel on the X axis so all animations should be comparable.

2. Fig. 2 shows stream lines of the "chaotic oscillator" which is defined by the field

$$\mathbf{v} = \begin{bmatrix} T_3\cos(T\,Y) + T_2\sin(T\,Z) \\ T_3\sin(T\,X) + T_1\cos(T\,Z) \\ T_2\cos(T\,X) + T_1\sin(T\,Y) \end{bmatrix}. \tag{125}$$

As already discussed in paper 257, this is a Beltrami field for $T_1 = T_2 = T_3$ but is not so chaotic as perhaps assumed, the stream lines move mainly along the coordinate axes, with mirrored symmetry between the planes.

3. The general Beltrami field can be written as

$$\mathbf{v} = \kappa\ \boldsymbol{\nabla}\times(\psi\mathbf{a}) + \boldsymbol{\nabla}\times\boldsymbol{\nabla}\times(\psi\mathbf{a}) \tag{126}$$

where ψ is an arbitrary function, κ is a constant and $\mathbf{a}$ is a constant vector. In paper 257 we discussed the example with

$$\psi = \frac{1}{L^3}XYZ, \tag{127}$$

$$\mathbf{a} = [0,0,1]. \tag{128}$$

The field is coplaner to the XY plane and gives planar streamlines of hyperbolic form (Fig. 3).

4. The solution of Rodrigues-Vaz is defined by

$$\mathbf{v} = \begin{bmatrix} -\left(\frac{\alpha\,\Omega\,y}{r^3} - \frac{\beta\,x\,z}{r^5}\right)\,C \\ -\left(-\frac{\beta\,y\,z}{r^5} - \frac{\alpha\,\Omega\,x}{r^3}\right)\,C \\ -\left(\frac{\beta\,\left(y^2+x^2\right)}{r^5} - \frac{2\,\alpha}{r^3}\right)\,C \end{bmatrix} \tag{129}$$

with constants C, Ω, α, β and radius function r. The field is a kind of spherical vortex field as can be seen from the field vector plot (Fig. 4). The vectors switch orientation over the Y direction by rotating into Z direction so a longitudinal component emerges. The projection of the three graphed Z planes into the XY plane (Fig. 5) shows that there is also a rotation in Z direction. At the origin the vector points exactly in Z direction therefore it is not visible in this graph. The streamlines (Fig. 6) of this Beltrami field show a chaotic vortex around the centre. This structure is confined by the inverse powers of r in the definition of $\mathbf{v}$. This reminds to the classical atomic model with electrons circulating around the nucleus.

5. Another known solution based on Bessel functions is the Lundquist solution

$$\mathbf{v} = \begin{bmatrix} J_1(\kappa r)\lambda e^{-\lambda Z} \\ J_1(\kappa r)\alpha e^{-\lambda Z} \\ J_1(\kappa r)e^{-\lambda Z} \end{bmatrix} \tag{130}$$

with

$$\kappa = \sqrt{\alpha^2 + \lambda^2} \tag{131}$$

and constants α and λ. The Lundquist function (for $Z > 0$) is graphed in Fig. 7 and initially behaves similar to the Bessel case discussed above. However the field shrinks with Z due to the exponential factor. Fig. 8 shows a projection into the XY plane. The vectors are always rotated by 45° against the radial direction. Longitudinal parts are not visible here as discussed for the Rodriguez-Vaz case. Outer streamlines (Fig. 9) go down to the region $Z < 0$, and here the exponential factor $\exp(-\lambda Z)$ gives an exponential growth, this is well recognizable in the second version of this animation on www.aias.us. λ can be assumed complex-valued as discussed by Reed, leading to oscillatory solutions, but then problems can arise in other parts of the field definition.

6. Finally we give some graphic examples for plane waves. Although these are well known, it is useful to recall certain features that not always are considered where plane waves are used. In ECE theory their most prominent appearance is in the vector potential of the free electromagnetic field, in cyclic cartesian coordinates:

$$\mathbf{A}_1 = \frac{A_0}{\sqrt{2}} \begin{bmatrix} e^{i\,(\omega\, t - \kappa\, Z)} \\ -i\; e^{i\,(\omega\, t - \kappa\, Z)} \\ 0 \end{bmatrix},\ \mathbf{A}_2 = \frac{A_0}{\sqrt{2}} \begin{bmatrix} e^{i\,(\omega\, t - \kappa\, Z)} \\ i\; e^{i\,(\omega\, t - \kappa\, Z)} \\ 0 \end{bmatrix},\ \mathbf{A}_3 = 0. \tag{132}$$

Their divergence is zero and the eigenvalue of the curl operator is κ or $-\kappa$, respectively. The plane wave can also be defined as real valued:

$$\mathbf{A}_1 = \frac{A_0}{\sqrt{2}} \begin{bmatrix} \cos(\omega\, t - \kappa\, Z) \\ -\sin(\omega\, t - \kappa\, Z) \\ 0 \end{bmatrix},\ \mathbf{A}_2 = \frac{A_0}{\sqrt{2}} \begin{bmatrix} \sin(\omega\, t - \kappa\, Z) \\ \cos(\omega\, t - \kappa\, Z) \\ 0 \end{bmatrix},\ \mathbf{A}_3 = 0 \tag{133}$$

and are Beltrami fields also, however with positive eigenvalues for $\mathbf{A}_1$ and $\mathbf{A}_2$. The real-valued plane waves are graphed as vector fields in Fig. 10 for a fixed instant of time $t = 0$. $\mathbf{A}_1$ and $\mathbf{A}_2$ are perpendicular to one another and define a rotating frame in Z direction. The streamlines in one plane are all parallel straight lines. To show a variation, they have been graphed in Fig. 11 for different starting points on the Z axis. Here the rotation of frames can be seen again.

Streamlines of plane waves are not very instructive concerning the physical meaning of these waves. It is more illustrative to show their time behaviour. We started with streamlines in the XY plane and computed their time evolution. The streamlines would remain in that plane so we added a Z component $v\ t$ to simulate a propagation in that direction as is the case for electromagnetic waves with $v = c$. Thus in Fig. 12 the trace of circularly polarized waves is obtained. Interestingly the waves are phase-shifted, although all starting points are at $Y = 0$.

In this paper we are considering plane wave in the context of Beltrami fields. As worked out the fields E, B and A are parallel. Therefore the components $\mathbf{A}_1$ and $\mathbf{A}_2$ do not demonstrate the behaviour of electric and magnetic fields of ordinary transversal electromagnetic fields which are phase-shifted by 90°. Reed[11] gives a very good explanation of this extraordinary case:

Every plane wave solution corresponds to two circularly polarized waves propagating oppositely to each other and combining to form a standing wave. This standing wave does not possess the standard power flow feature of linearly- or circularly-polarized waves with $E \perp B$, since the combined Poynting vectors of the circularly-polarized waves cancel each other similar to the situation we met earlier in connection with Beltrami plasma vortex filaments. Essentially, the combination of these two waves produces a standing wave propagating non-zero magnetic helicity. In the book by Marsh [12] the relationship is shown between the helicity and energy densities for this wave as well, as the very interesting fact that any magnetostatic solution to the FFMF equations can be used to construct a solution to Maxwell´s equations with $E \| B$.

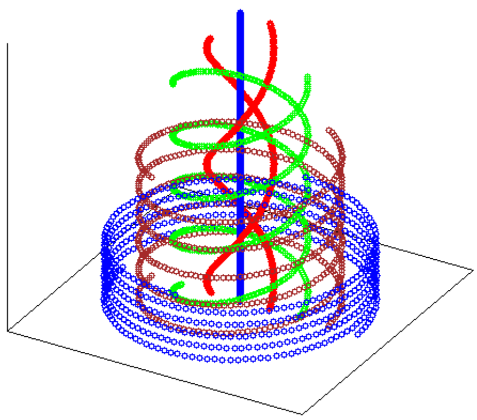

Figure 1: Streamlines of the Bessel function solution.

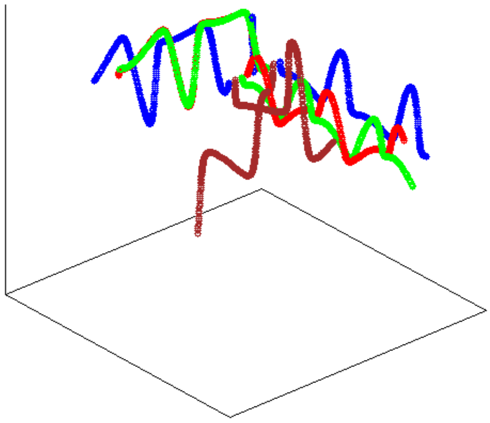

Figure 2: Streamlines of the chaotic oscillator.

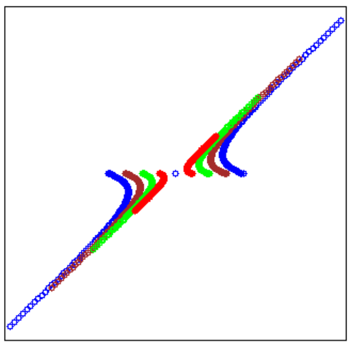

Figure 3: Streamlines of the general solution with $\psi = \frac{1}{L^3} XYZ$.

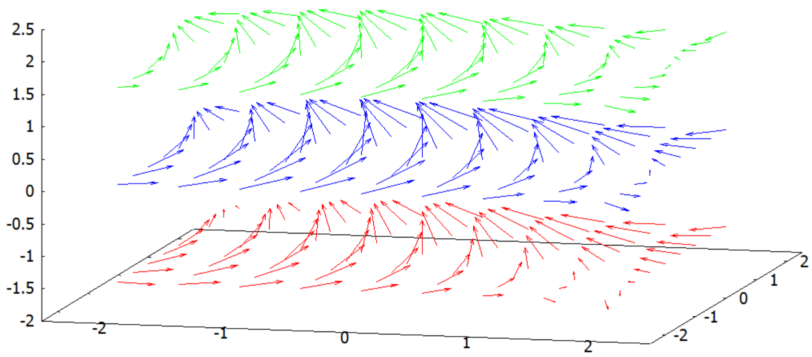

Figure 4: Rodrigues-Vaz solution.

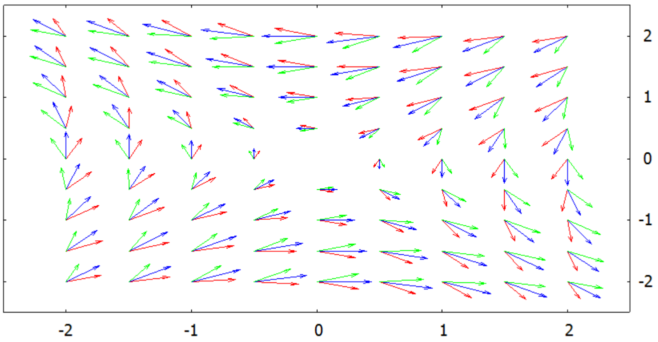

Figure 5: Rodrigues-Vaz solution, projected to XY plane.

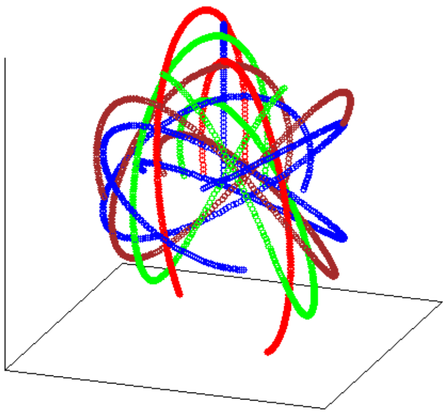

Figure 6: Streamlines of Rodrigues-Vaz solution.

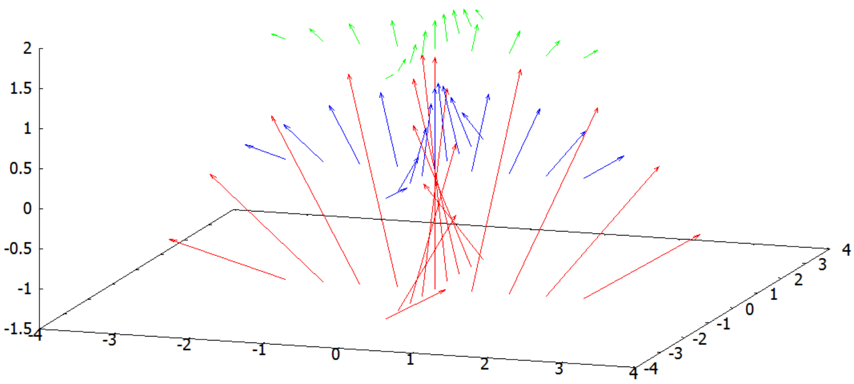

Figure 7: Lundquist solution.

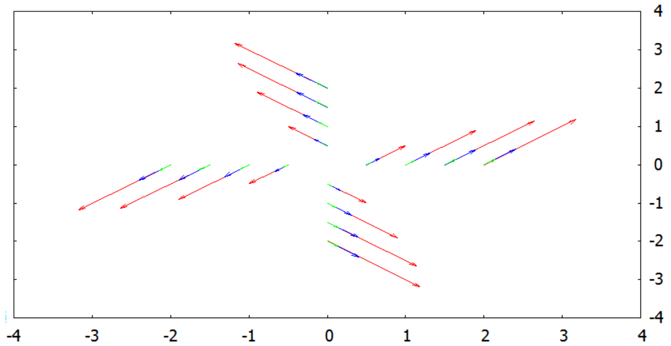

Figure 8: Lundquist solution, projected to XY plane.

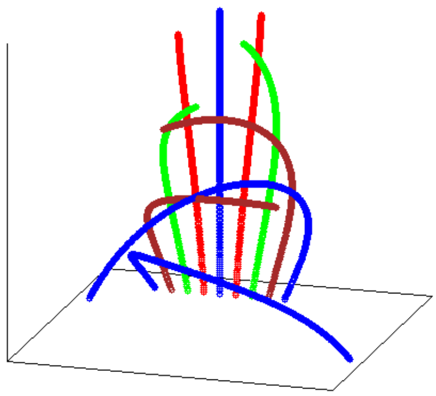

Figure 9: Streamlines of Lundquist solution.

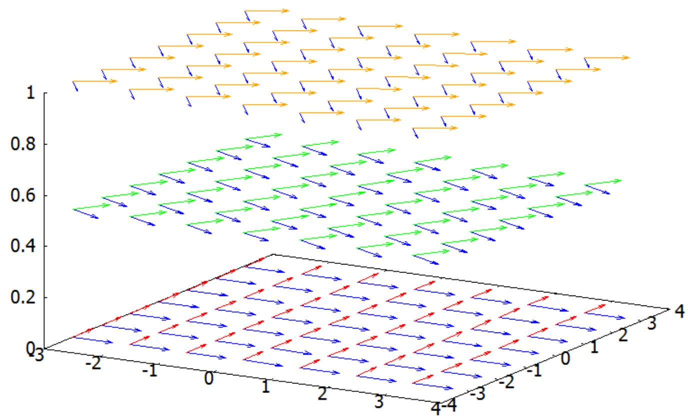

Figure 10: Plane wave field, $\mathbf{A}_1$ and $\mathbf{A}_2$.

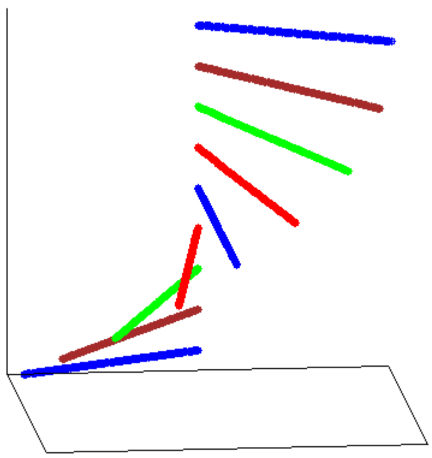

Figure 11: Streamlines of plane waves.

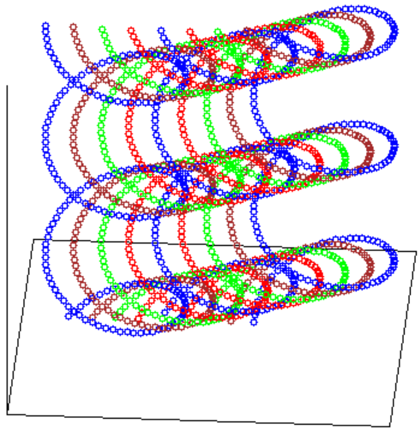

Figure 12: Time evolution of points transported by plane waves.

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