ANALYTICAL SOLUTION OF THE N PARTICLE GRAVITATIONAL PROBLEM: STOKES' THEOREM AND ORBITAL CIRCULATION.

by

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ABSTRACT

The N particle gravitational problem is solved analytically in the pairwise additive approximation. The solution is illustrated for three interacting particles, and Stokes' Theorem used to introduce the concept of orbital circulation. It is shown numerically that the orbital circulation is non-zero for all conical sections and precessing conical sections with the exception of circular orbits. The orbital circulation may therefore be used to characterize all orbits known in cosmology.

Keywords: Classical limit of ECE theory, solution of the N particle gravitational problem, orbital circulation.

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1. INTRODUCTION

Recently in this series of papers {1 - 10} on the applications of ECE theory the Einsteinian general relativity (EGR) has been refuted in many ways and replaced by a relativity based on the constrained Minkowski metric. In the classical limit of this new relativity it has been shown that all the features of planetary precession can be explained straightforwardly with the equation of the precessing conical section, and a myriad of new properties discovered in terms of the precession constant x. All known orbits in cosmology have been shown to be explicable in terms of x and the equation of the precessing conical section. In immediately preceding papers the investigation of the classical limit of the new relativity has been extended to the multi particle gravitational problem, thought to have no known analytical solution, and in the preceding paper a new form of Kepler's third law inferred for a precessing orbit.

In Section 2 of this paper it is shown that the N particle gravitational problem can be solved analytically in a relatively straightforward way given the usual form of the starting lagrangian for the problem. This is a four hundred year old problem in cosmology, and up to now it has been thought to have only specialized solutions discovered by Euler, Lagrange, Poincare and others. Stokes' Theorem is used to develop the solution for three interacting particles, and a new concept developed of orbital swirl or circulation. In Section 3 some of the results are illustrated graphically.
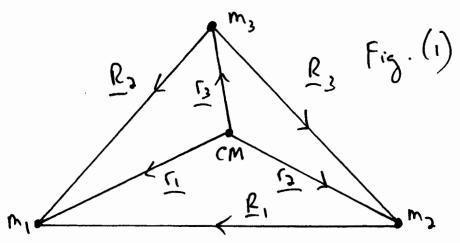
2. SOLUTION AND CONCEPT OF ORBITAL CIRCULATION

Consider three masses interacting simultaneously with the following lagrangian

$$\mathcal{L} = \frac{1}{2}\left(m_1|\dot{\underline{r}}_1|^2 + m_2|\dot{\underline{r}}_2|^2 + m_3|\dot{\underline{r}}_3|^2\right)$$
$$+ \frac{m_1 m_2 \, G}{|\underline{r}_1 - \underline{r}_2|} + \frac{m_1 m_3 \, G}{|\underline{r}_1 - \underline{r}_3|} + \frac{m_2 m_3 \, G}{|\underline{r}_2 - \underline{r}_3|} \qquad - (1)$$

This is the standard format of the lagrangian for what is known as the three particle gravitational problem in Newtonian dynamics. Here m are the masses, G is Newton's constant, and the coordinates are defined in Fig. (1).

Fig. (1)

The centre of mass of the three particle system is defined to be at the origin of the coordinate system. So:

$$m_1 \underline{r}_1 + m_2 \underline{r}_2 + m_3 \underline{r}_3 = \underline{0} \qquad - (2)$$

As in Fig. (1):

$$\underline{R}_1 = \underline{r}_1 - \underline{r}_2 \qquad - (3)$$
$$\underline{R}_2 = \underline{r}_1 - \underline{r}_3 \qquad - (4)$$
$$\underline{R}_3 = \underline{r}_2 - \underline{r}_3 . \qquad - (5)$$

Therefore the lagrangian is:

$$\mathcal{L} = \frac{1}{2}\left(m_1|\dot{\underline{r}}_1|^2 + m_2|\dot{\underline{r}}_2|^2 + m_3|\dot{\underline{r}}_3|^2\right) \qquad -(6)$$
$$+ \frac{m_1 m_2 G}{R_1} + \frac{m_1 m_3 G}{R_2} + \frac{m_2 m_3 G}{R_3}.$$

From Eqs. (2) and (3):

$$m_1 \underline{r}_1 + m_2\left(\underline{r}_1 - \underline{R}_1\right) + m_3 \underline{r}_3 = \underline{0} \qquad -(7)$$

i.e.

$$\underline{r} = -\frac{\left(m_1 \underline{R}_1 + m_3 \underline{r}_3\right)}{m_1 + m_2} \qquad -(8)$$

Similarly:

$$m_1 \underline{r}_1 + m_2\left(\underline{r}_1 - \underline{R}_1\right) + m_3 \underline{r}_3 = \underline{0} \qquad -(9)$$

i.e.:

$$\underline{r}_1 = \frac{m_2 \underline{R}_1 - m_3 \underline{r}_3}{m_1 + m_2} \qquad -(10)$$

Therefore:

$$r_1^2 = \frac{m_2^2 R_1^2 + m_3^2 r_3^2 - 2 m_2 m_3 \underline{R}_1 \cdot \underline{r}_3}{(m_1 + m_2)^2} \qquad -(11)$$

and

$$r_2^2 = \frac{m_1^2 R_1^2 + m_3^2 r_3^2 + 2 m_1 m_3 \underline{R}_1 \cdot \underline{r}_3}{(m_1 + m_2)^2} \qquad -(12)$$

Solving Eqs. (11) and (12) gives the reduced lagrangian:

$$\mathcal{L} = \frac{1}{2}\mu|\underline{\dot{R}}_1|^2 + m_3\left(1 + \frac{m_3}{m_1 + m_2}\right)|\underline{\dot{s}}_3|^2$$
$$+ \frac{m_1 m_2 G}{R_1} + \frac{m_1 m_3 G}{R_2} + \frac{m_2 m_3 G}{R_3} \qquad -(13)$$

where the reduced mass is:

$$\mu = \frac{m_1 m_2}{m_1 + m_2} \qquad -(14)$$

In plane polar coordinates:

$$|\underline{\dot{R}}_1|^2 = \dot{R}_1^2 + R_1^2 \dot{\theta}_1^2 \qquad -(15)$$

Now consider the Euler Lagrange equations:

$$\frac{\partial \mathcal{L}}{\partial R_1} = \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{R}_1} \qquad -(16)$$

and

$$\frac{\partial \mathcal{L}}{\partial \theta_1} = \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{\theta}_1} \qquad -(17)$$

with the lagrangian (13). Ths solution of Eqs. (16) and (17) is well known {11} to be the elliptical orbit:

$$R_1 = \frac{\alpha_1}{1 + \epsilon_1 \cos\theta_1} \qquad -(18)$$

where α_1 is the semi right latitude and ϵ_1 is the eccentricity. Similarly:

$$R_2 = \frac{\alpha_2}{1 + \epsilon_2 \cos\theta_2} \qquad -(19)$$

and

$$R_3 = \frac{\alpha_3}{1 + \epsilon_3 \cos\theta_3} \quad\quad -(20)$$

where:

$$\underline{R_2} = \underline{R_1} + \underline{R_3} . \quad -(21)$$

This is the same result as in previous work {1 - 10} and obtained with a different method.

Therefore it follows that the general solution of the N particle problem is:

$$R_i = \frac{\alpha_i}{1 + \epsilon_i \cos\theta_i} , \quad i = 1, .., N \quad -(22) .$$

given the lagrangian (13) extended to N particles. For N masses in a plane the constraint

(21) is extended to N masses. For precessing orbits {1 - 10} the general solution is

$$R_i = \frac{\alpha_i}{1 + \epsilon_i \cos(x_i \theta_i)} , \quad i = 1, \ldots, N \quad -(23)$$

where x_i is the precession factor for each orbit. Therefore the N particle gravitational

problem has been solved for the first time.

Straightforward application of Stokes' Theorem gives:

$$\oint \underline{R_i} \cdot d\underline{R_i} = \int \underline{S_i} \cdot \underline{n} \, dA_i \quad -(24)$$

where $\underline{S_i}$ is the orbital circulation:

$$S_i = \underline{V} \times \underline{R_i} = -\frac{1}{R_i} \frac{\partial R_i}{\partial \theta_i} \underline{k}. \quad -(25)$$

From Eq. (22) in the Newtonian limit:

$$\frac{\partial R_i}{\partial \theta_i} = \frac{\epsilon_i}{d_i} R_i^2 \sin\theta_i \quad -(26)$$

so:

$$S_i = -\frac{\epsilon_i \sin\theta_i}{1 + \epsilon_i \cos\theta_i}. \quad -(27)$$

The orbital circulation is a new concept in cosmology and some examples are graphed in Section 3. For precessing orbits:

$$S_i = -\frac{\epsilon_i x_i \sin(x_i \theta_i)}{1 + \epsilon_i \cos(x_i \theta_i)} \quad -(28)$$

in which the gravitational potential is the new universal law of gravitation:

$$U(R) = -\frac{mMGx^2}{R} - \frac{L^2}{2mR^2}(1-x^2) \quad -(29)$$

in which the total angular momentum is the constant:

$$L = \frac{mM}{m+M} R^2 \frac{d\theta}{dt}. \quad -(30)$$

In general for any curve in a plane {11}:

$$dA_i = \frac{1}{2} R_i^2 \, d\theta_i \quad - (31)$$

so:

$$\oint \underline{R_i} \cdot d\underline{R_i} = \frac{1}{2} \int S_i \underline{k} \cdot \underline{n} \, R_i^2 \, d\theta_i \quad - (32)$$

Assuming:

$$\underline{k} \cdot \underline{n} = 1 \quad - (33)$$

then:

$$\oint \underline{R_i} \cdot d\underline{R_i} = -\frac{1}{2} \int S_i R_i^2 \, d\theta_i = \epsilon_i \alpha_i^2 \int \frac{\sin\theta_i \, d\theta_i}{(1 + \epsilon_i \cos\theta_i)^3} \quad - (34)$$

This integral was evaluated numerically and gives the result:

$$\oint \underline{R_i} \cdot d\underline{R_i} = \frac{1}{2} R_i^2 \quad - (35)$$

For precessing orbits:

$$\oint \underline{R_i} \cdot d\underline{R_i} = \epsilon_i \alpha_i^2 x_i \int \frac{\sin(x_i \theta_i) \, d\theta_i}{(1 + \epsilon_i \cos(x_i \theta_i))^3} \quad - (36)$$

In the three particle problem:

$$\oint \underline{R_2} \cdot d\underline{R_2} = \oint (\underline{R_1} + \underline{R_3}) \cdot d\underline{R_2} \quad - (37)$$

so the circulation vectors are additive as follows:

$$\underline{S_2} = \underline{S_1} + \underline{S_3} \quad - (38)$$

providing the constraint on the solution (27):

$$\frac{\epsilon_2 \sin \theta_2}{1 + \epsilon_2 \cos \theta_2} = \frac{\epsilon_1 \sin \theta_1}{1 + \epsilon_1 \cos \theta_1} + \frac{\epsilon_3 \sin \theta_3}{1 + \epsilon_3 \cos \theta_3}$$

$$- (39)$$

For each orbit in the Newtonian limit the elapsed times are:

$$t_i = \left(1 - \epsilon_i^2\right)^{3/2} \left(\frac{\tau_i}{2\pi}\right) \Big/ \frac{\mu\theta_i}{\left(1 + \epsilon_i \cos \theta_i\right)^2} \quad - (40)$$

where the eccentricities are:

$$\epsilon_i = \left(1 - \left(\frac{a_i}{b_i}\right)^2\right)^{1/2}, \quad - (41)$$

and where τ_i are the times taken for each orbit to transcribe 2π radians.

The orbital circulation seems to be a new concept of cosmology, and may be used to characterize every orbit. It is non-zero for every orbit in general, with the exception of a circular orbit. For a circular orbit:

$$\underline{R} = X \underline{i} + Y \underline{j} \quad - (42)$$

$$d\underline{R} = dX \underline{i} + dY \underline{j} \quad - (43)$$

therefore:

$$\oint \underline{R} \cdot d\underline{R} = \int_0^{2\pi} (X dX + Y dY) \quad - (44)$$

In the case of the circle:

$$X = R\cos\theta \; ; \; Y = R\sin\theta, \quad -(45)$$

$$dX = -R\sin\theta \, d\theta \quad -(46)$$

$$dY = R\cos\theta \, d\theta \quad -(47)$$

therefore:

$$\int_0^{2\pi}(X dX + Y dY) = \int_0^{2\pi} R\cos\theta\left(-R\sin\theta\right)d\theta + R\sin\theta\left(R\cos\theta\right)d\theta$$

$$= 0 \qquad -(48)$$

The contour integral around the circumference of a circle is zero. The curl is:

$$\nabla \times \underline{R} = \left(\frac{\partial Y}{\partial X} - \frac{\partial X}{\partial Y}\right)\underline{k} = \underline{0} \quad -(49)$$

so Stokes' Theorem is verified. In cylindrical polar coordinates:

$$\underline{R} = R\underline{e}_r \quad -(50)$$

where:

$$\underline{e}_r = \underline{i}\cos\theta + \underline{j}\sin\theta \quad -(51)$$

and:

$$\nabla \times \underline{R} = -\frac{1}{R}\frac{\partial R}{\partial\theta}\underline{k} \cdot -(52)$$

The circle is defined by the conical section equation:

$$R = \frac{\alpha}{1 + \epsilon \cos \theta} \quad -(53)$$

with:

$$\epsilon = 0. \quad -(54)$$

So:

$$R = \alpha = \text{constant} \quad -(55)$$

and

$$\frac{\partial R}{\partial \theta} = 0, \quad -(56)$$

therefore

$$\nabla \times \underline{R} = \underline{0} \quad -(57)$$

Q.E.D.

3. GRAPHICAL ILLUSTRATIONS.

Section by Dr. Horst Eckardt

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3   Graphical illustrations

The orbital circulation S of Eq.(28) is demonstrated in Fig. 1 for three values of x. This function oscillates between negative and positive values. In Fig. 1 the elliptic radius r has been added for comparison. It can be seen that S crosses zero where the ellipse takes its minimum and maximum radial values. This has been shown in a polar plot in Fig. 2. Since negative values are plotted as positive values rotated by 180 degrees, two ellipsoidal structures appear for S in Fig. 2. Obviously S is not symmetric for positive and negative values, the ellipsoidal structures are distinct.

The circulation integral of Eq.(36) can be solved to be

$$\oint \mathbf{R}_i \cdot d\mathbf{R}_i = \frac{\alpha_i{}^2}{2\left(1 + \epsilon_i \cos\left(x_i\,\theta\right)\right)^2}. \tag{58}$$

This is a positive function and takes its maximum where $r(\theta)$ is maximal too, see Fig. 3.

For hyperbolas, the S function shows poles as graphed in Fig. 4. Correspondingly, the circulation integral approaches infinity at the same values (Fig. 5). The effect of x is to stretch the angular axis in all cases.

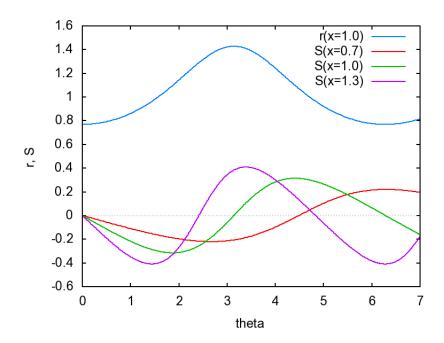*email: emyrone@aol.com

†email: horsteck@aol.com

Figure 1: Orbital circulation S for $\epsilon = 0.3$ and different x values, compared with elliptic radius r.
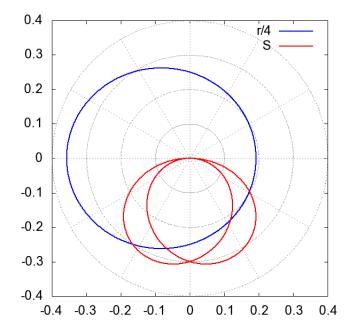

Figure 2: Polar plot of orbital circulation S for $\epsilon = 0.3$, $x = 1$, compared with scaled elliptic radius $r/4$.
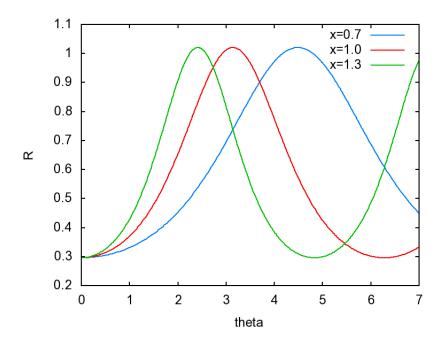
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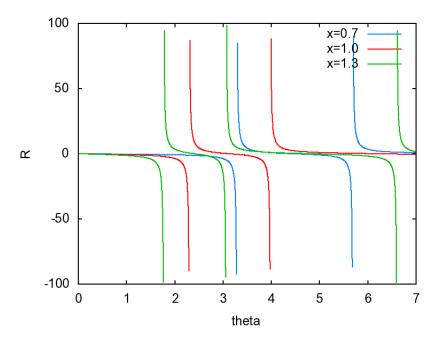Figure 3: Circulation integral for $\alpha = 1$, $\epsilon = 0.3$ and different x values.

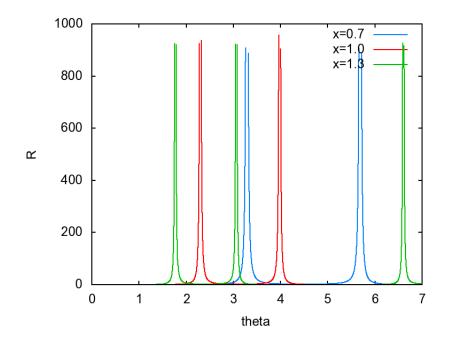Figure 4: Orbital circulation S for a hyperbola with $\epsilon = 1.5$.

3

Figure 5: Circulation integral for a hyperbola with $\alpha = 1$, $\epsilon = 1.5$.

ACKNOWLEDGMENTS

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