Chapter 13

Theory of SU(2) Quantum Electrodynamics

by

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Abstract

The ECE equation of the fermion is simplified to a format in which it is clear that the wave function is made up of components of a tetrad. The fundamental ECE hypothesis is applied to find an equation for quantum electrodynamics in an SU(2) representation space. The interaction of a fermion with electromagnetic radiation can therefore be developed using two simultaneous SU(2) equations, one for the quantized fermion field, one for the photon. This is a fully quantized development in the limit of special relativity. The procedure is also valid in general relativity, because these simultaneous equations are factorizations of two ECE wave equations. The procedure is illustrated by deriving ESR and NMR from the new equations.

Keywords: Einstein Cartan Evans (ECE) theory, ECE equation of the fermion, quantum electrodynamics, ESR and NMR.

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13.1 Introduction

Recently in this series of papers [1]- [10] the equation of the fermion was derived from ECE unified field theory. It was shown that the fermion equation can be expressed in terms of two by two matrices, thus eliminating the need for the four by four Dirac matrices [11]. In this paper that procedure is extended to quantum electrodynamics using the fundamental ECE hypothesis:

$$A^a_\mu = \mathrm{A}^{(0)} q^a_\mu \tag{13.1}$$

where A^a_μ is the potential field and q^a_μ is the Cartan tetrad. Here $c\mathrm{A}^{(0)}$ in S.I. units as a primordial or background voltage which pervades the vacuum in ECE theory [12]. In SU(2) representation space the Pauli matrices are 2 x 2 tetrads (see background notes to this paper on www.aias.us), and in SU(3) representation space the Gell-Mann matrices are 3 x 3 tetrads. The wave-function of the ECE fermion equation was shown in papers 129 and 130 of this series to be made up of tetrad components. The ECE fermion equation was developed in terms of four simultaneous equations linking tetrad components. In Section 2 it is shown that this development can be simplified, so that the ECE fermion equation becomes:

$$\sigma^\mu p_\mu \Phi^R = mc\sigma^0 \Phi^L \tag{13.2}$$

in momentum representation [11]. Here:

$$\Phi^R = [q^R_1 q^R_2], \tag{13.3}$$

$$\Phi^L = [q^L_1 q^L_2] \tag{13.4}$$

where q^R_1 and q^R_2 are components of the right handed tetrad, and where q^L_1 and q^L_2 are components of the left handed tetrad. In Eq. (13.2), σ^μ is a four vector made up of Pauli matrices, p_μ is the four momentum of the fermion, m is its mass, and c is the vacuum speed of light. It is shown that the ECE hypothesis (13.1) may be applied to give a fully quantized equation of electrodynamics, and therefore an equation of the photon with mass. The field of the quantized potential A^a_μ may be found from the quantized Cartan torsion. The quantized interaction of the electron and photon, for example, may be developed as in Section 3 by solving the SU(2) equations of the particles simultaneously. The procedure is based entirely on geometry, and rejects indeterminacy and the Dirac sea in favour of objective and deterministic physics. Finally the procedure is illustrated by deriving the usual properties such as the Zeeman effect, ESR and NMR.

13.2 Equations of the Fermion and Photon

In paper 130 the ECE equation of the fermion of mass m was developed as four simultaneous equations:

$$(\sigma^0 p_0 - \boldsymbol{\sigma} \cdot \mathbf{p}) q^R_1 = mc\sigma^0 q^L_1 \tag{13.5}$$

$$(\sigma^0 p_0 - \boldsymbol{\sigma} \cdot \mathbf{p}) q_2^R = mc\sigma^0 q_2^L \tag{13.6}$$

$$(\sigma^0 p_0 + \boldsymbol{\sigma} \cdot \mathbf{p}) q_1^L = mc\sigma^0 q_1^R \tag{13.7}$$

$$(\sigma^0 p_0 + \boldsymbol{\sigma} \cdot \mathbf{p}) q_2^L = mc\sigma^0 q_2^R \tag{13.8}$$

where the Pauli matrix four vector is:

$$\sigma^\mu = (\sigma^0, \sigma^1, \sigma^2, \sigma^3) \tag{13.9}$$

and the momentum four vector is:

$$p_\mu = (p_0, -\mathbf{p}). \tag{13.10}$$

These are equations in the components of the tetrad:

$$q_\mu^a = \begin{bmatrix} q_1^R & q_2^R \\ q_1^L & q_2^L \end{bmatrix}. \tag{13.11}$$

The Pauli matrices are basis elements and are related by the SU(2) Lie algebra:

$$[\frac{\sigma^1}{2}, \frac{\sigma^2}{2}] = i\frac{\sigma^3}{2} \tag{13.12}$$

etc. as is well known. Eqs. (13.5) and (13.6) can be written as:

$$\sigma^\mu p_\mu q_1^R = mc\sigma^0 q_1^L, \tag{13.13}$$

$$\sigma^\mu p_\mu q_2^R = mc\sigma^0 q_2^L \tag{13.14}$$

which can be condensed into:

$$\sigma^\mu p_\mu \Phi^R = mc\sigma^0 \Phi^L \tag{13.15}$$

where:

$$\Phi^R = [q_1^R q_2^R], \tag{13.16}$$

$$\Phi^L = [q_1^L q_2^L]. \tag{13.17}$$

Using the operator equivalence:

$$p_\mu = i\hbar\partial_\mu \tag{13.18}$$

Eq. (13.15) is a first order differential equation of quantum mechanics:

$$i\sigma^\mu \partial_\mu \Phi^R = (\frac{mc}{\hbar})\ \sigma^0 \Phi^L. \tag{13.19}$$

Eqs. (13.7) and (13.8) follow from eqs. (13.5) and (13.6), so the latter are sufficient to describe the fermion. This procedure greatly simplifies and clarifies

Dirac's description of the fermion [11] and explains the existence of the fermion in terms of geometry. Multiply both sides of Eq. (13.5) by $(\sigma^0 p_0 + \boldsymbol{\sigma} \cdot \mathbf{p})$:

$$(\sigma^0 p_0 - \boldsymbol{\sigma} \cdot \mathbf{p})(\sigma^0 p_0 + \boldsymbol{\sigma} \cdot \mathbf{p}) q_1^R = mc\sigma^0(\sigma^0 p_0 + \boldsymbol{\sigma} \cdot \mathbf{p}) q_1^L. \tag{13.20}$$

Use:

$$\sigma^0 p_0 \sigma^0 p_0 = (\sigma^0)^2 p_0^2 \tag{13.21}$$

and:

$$(\ \sigma \cdot \mathbf{p})(\boldsymbol{\sigma} \cdot \mathbf{p}) = \sigma^0 \mathbf{p} \cdot \mathbf{p} + i\boldsymbol{\sigma} \cdot \mathbf{p} \times \mathbf{p}. \tag{13.22}$$

If the momentum $\mathbf{p}$ is considered to be real valued, then:

$$\mathbf{p} \times \mathbf{p} = \mathbf{0}. \tag{13.23}$$

So we obtain:

$$(\boldsymbol{\sigma} \cdot \mathbf{p})(\boldsymbol{\sigma} \cdot \mathbf{p}) = \sigma^0 \mathbf{p} \cdot \mathbf{p}. \tag{13.24}$$

However, the Einstein energy equation [11] of special relativity is:

$$p^\mu p_\mu = {p_0}^2 - \mathbf{p} \cdot \mathbf{p} = m^2 c^2 \tag{13.25}$$

so Eq. (13.20) becomes:

$$(\sigma^0 p_0 + \boldsymbol{\sigma} \cdot \mathbf{p}) q_1^L = \sigma^0\ m\, c\, q_1^R \tag{13.26}$$

which is Eq.(13.7), Q.E.D.

All the information about the fermion is therefore contained in Eq. (13.19). This procedure simplifies the equation of the fermion obtained in paper 130.The anti-fermion is obtained straightforwardly as in paper 130 (www.aias.us). Eq. (13.19) is an equation linking two tetrad components, and is a factorization of the ECE wave equation [1] - [10] in the limit where the fermion is free of other fields:

$$(\Box + (c/\hbar)^2) q_\mu^a = 0 \tag{13.27}$$

where m is the fermion mass. Here h is the reduced Planck constant and c is the speed of light in vacuo. The wave function of Eq. (13.27) is the tetrad, a basic characteristic of ECE theory, in which the unified field is developed from geometry. Eq. (13.19) may be thought of as a "factorization" of Eq. (13.27). The fundamental hypothesis of ECE theory is that the potential of the unified field of force of physics is proportional to the Cartan tetrad of geometry:

$$A_\mu^a = \mathrm{A}^{(0)} q_\mu^a. \tag{13.28}$$

The quantity c $\mathrm{A}^{(0)}$ is by postulate the primordial voltage of the ECE vacuum. This is responsible for the radiative corrections as in papers such as 85 of www.aias.us. The primordial voltage may be used for practical applications as described in papers such as 134 of the ECE series on www.aias.us. If the hypothesis (13.28) is applied to Eq. (13.19), the following equation is obtained:

$$i\sigma^\mu \partial_\mu A^R = (\frac{mc}{\hbar})\sigma^0 \mathrm{A}^L. \tag{13.29}$$

If A^a_μ is negative under charge conjugation symmetry [11], Eq. (13.29) may be interpreted as an equation of the photon, whose wave equation is the ECE generalization of the Proca equation:

$$(\Box + (c/\hbar)^2)A^a_\mu = 0. \tag{13.30}$$

In Eqs. (13.29) and (13.30) m is the mass of the photon [1]- [10]. In the standard physics, the fermion is interpreted as obeying Fermi-Dirac statistics and the photon (a boson), as obeying Bose-Einstein statistics. In Eq. (13.29) however, the electromagnetic potential is developed in SU(2) representation space using the tetrad:

$$A^a_\mu = \begin{bmatrix} A_1^R & A_2^R \\ A_1^L & A_2^L \end{bmatrix}. \tag{13.31}$$

This is an example of the fact that in ECE, the field is unified, and may be developed in any representation space. Therefore there may be an SU(2) electrodynamics, and SU(3) electrodynamics and so on. The standard development of electrodynamics still uses an obsolete Minkowski spacetime, whose space part uses Cartesian unit vectors, an O(3) representation space. Much more information about electrodynamics is available in the ECE theory. This conclusion is a direct consequence of geometry, specifically of the fact that a wave equation:

$$\Box q^a_\mu = R q^a_\mu, \quad R = -(\frac{mc}{\hbar})^2 \tag{13.32}$$

can be reduced to the form:

$$i\sigma^\mu \partial_\mu \Phi^R = |R|^{1/2} \sigma^0 \Phi^L \tag{13.33}$$

as shown here for the first time. This inference opens up many new possibilities in field theory.

13.3 Interaction of the Fermion and Photon

This interaction may now be described by the following four equations, the first two for the fermion and the second two for the photon:

$$(\sigma^0 p_0 - \boldsymbol{\sigma} \cdot \mathbf{p})q_1^R = mc\sigma^0 q_1^L \tag{13.34}$$

$$(\sigma^0 p_0 - \boldsymbol{\sigma} \cdot \mathbf{p})q_2^R = mc\sigma^0 q_2^L \tag{13.35}$$

$$(\sigma^0 p_0 + \boldsymbol{\sigma} \cdot \mathbf{p})A_1^L = mc\sigma^0 A_1^R \tag{13.36}$$

$$(\sigma^0 p_0 + \boldsymbol{\sigma} \cdot \mathbf{p})A_2^L = mc\sigma^0 A_2^R \tag{13.37}$$

These must be solved simultaneously. Using the minimal prescription:

$$p_\mu A_1^L = A_\mu q_1^L \tag{13.38}$$

and so on, so Eqs. (13.36) and (13.37) may be written as:

$$(\sigma^0 A_0 + \boldsymbol{\sigma} \cdot \mathbf{A})q_1^L = A_0 \sigma^0 q_1^R, \tag{13.39}$$

$$(\sigma^0 A_0 + \boldsymbol{\sigma} \cdot \mathbf{A})q_2^L = A_0 \sigma^0 q_2^R. \tag{13.40}$$

Multiply both sides of Eq. (13.34) by $(\sigma^0 A_0 + \boldsymbol{\sigma} \cdot \mathbf{A})$:

$$(\sigma^0 p_0 - \boldsymbol{\sigma} \cdot \mathbf{p})(\sigma^0 A_0 + \boldsymbol{\sigma} \cdot \mathbf{A})\ q_1^L = (\sigma^0 A_0 + \boldsymbol{\sigma} \cdot \mathbf{A})mc\sigma^0 q_1^L \tag{13.41}$$

and develop the relevant term as follows:

$$(\boldsymbol{\sigma} \cdot \mathbf{p})(\boldsymbol{\sigma}.\mathbf{A})q_1^L = \ -ih(\boldsymbol{\sigma} \cdot \nabla)((\boldsymbol{\sigma} \cdot \mathbf{A})q_1^L) = -i\hbar\boldsymbol{\sigma} \cdot \boldsymbol{\nabla} \times \mathbf{A} q_1^L + ih\boldsymbol{\sigma} \cdot (\mathbf{A} \times \boldsymbol{\nabla} q_1^L). \tag{13.42}$$

As is well known, the existence of ESR, NMR and MRI, important industries, depends on the term i h $\boldsymbol{\sigma}$. B . In this development, the magnetic flux density B has been identified as:

$$\mathbf{B} := \boldsymbol{\nabla} \times \mathbf{A}. \tag{13.43}$$

More generally [1]- [10], the magnetic flux density contains a spin connection term which emerges from the relation between the ECE field and the ECE potential. The background to these considerations are given in the notes accompanying this paper (number 135). These notes for each paper are posted in full detail on www.aias.us. The development of a quantized electromagnetic field from the quantized potential of Eq. (13.29) proceeds through the Cartan structure equation:

$$F^a = d \wedge A^a + \omega^a_b \wedge A^b \tag{13.44}$$

from which it is seen that the electromagnetic field is quantized if the potential is quantized. In the development of this paper, the minimal prescription has been used to relate linear momentum and the potential.

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