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A simple scheme for calculating the resonance effect

In contrast to the derivation in the numerical paper, we derive the resonance computation scheme from the E and B fields instead of the potential A. The ECE equations for the electric and magnetic field with gravitational interaction read

$$\boldsymbol{\nabla} \times \mathbf{E}^a + \frac{\partial \mathbf{B}^a}{\partial t} = \mu_0 \mathbf{j}^a \qquad (1)$$

$$\boldsymbol{\nabla} \times \mathbf{B}^a - \frac{1}{c^2}\frac{\partial \mathbf{E}^a}{\partial t} = \mu_0 \mathbf{J}^a \qquad (2)$$

with the well-known expressions for the homogeneous current and inhomogeneous current:

$$\mathbf{j}^a = \frac{1}{\mu_0}\left(R^a{}_b \wedge A^b - \omega^a{}_b \wedge F^b \right)$$

$$\mathbf{J}^a = \frac{1}{\mu_0}\left(\tilde{R}^a{}_b \wedge A^b - \omega^a{}_b \wedge \tilde{F}^b \right)$$

We assume a harmonic time dependence as described in the numerical paper:

$$\left.\begin{aligned} \mathbf{j}^a &= \mathbf{j}^a(\mathbf{x})\, e^{-i\omega t} \\ \mathbf{J}^a &= \mathbf{J}^a(\mathbf{x})\, e^{-i\omega t} \\ \mathbf{E}^a &= \mathbf{E}^a(\mathbf{x})\, e^{-i\omega t} \\ \mathbf{B}^a &= \mathbf{B}^a(\mathbf{x})\, e^{-i\omega t} \end{aligned}\right\} \qquad (3)$$

From eqs. (1) and (2) then follows

$$\boldsymbol{\nabla} \times \mathbf{E}^a - i\omega \mathbf{B}^a = \mu_0 \mathbf{j}^a \qquad (4)$$

$$\boldsymbol{\nabla} \times \mathbf{B}^a + i\frac{\omega}{c^2} \mathbf{E}^a = \mu_0 \mathbf{J}^a \qquad (5)$$

This is in analogy to deriving the Helmholtz wave equations from Maxwell's equations. Now we have stationary equations for E and B. To study the resonance behaviour we proceed again as in the numerical paper. We assume a fixed j and J and vary the frequency. For each frequency we obtain a maximum of the energy density

$$w_{max} = \frac{1}{2} \max_{\underline{x} \in V} \left\{ |\underline{E}^a|^2 + |\underline{B}^a|^2 \right\} \qquad (6)$$

in the definition volume V. The resonance occurs, where w_{max} (ω) is minimal, i.e. the same inhomogeneous current is obtained by a minimal field.

The numerical computation of E and B can be performed by FEA or a finite difference scheme. In case of the latter a simple iteration scheme for successive overrelaxation can be derived from eqs. (4), (5):

$$\underline{B}^a = \frac{1}{i\omega} \left(\underline{\nabla} \times \underline{E}^a - \mu_0 \underline{j}^a \right) \qquad (7)$$

$$\underline{E}^a = \frac{c^2}{i\omega} \left(-\underline{\nabla} \times \underline{B}^a + \mu_0 \underline{J}^a \right) \qquad (8)$$

With

$$\underline{B}^a = \mu_0 \underline{H}^a \qquad (9)$$

we obtain nearly symmetric equations

$$\underline{H}^a = \frac{1}{i\omega\mu_0} \left(\underline{\nabla} \times \underline{E}^a - \mu_0 \underline{j}^a \right) \qquad (10)$$

$$\underline{E}^a = \frac{1}{i\omega\varepsilon_0} \left(-\underline{\nabla} \times \underline{H}^a + \underline{J}^a \right) \qquad (11)$$

Using H rather than B has the advantage that both fields E and H lie in the same numerical order of magnitude, which gives more numerical stability.

Eqs. (10) and (11) represent an iteration scheme. The right hand side is computed by the values of E and H from iteration n, leading to values for iteration n+1 on the left.

Alternatively the dielectric formulation for eqs. (1) and (2) can be used as derived in paper 50:

$$\underline{\nabla} \times \left(\varepsilon_{r1} \underline{E}^a \right) + \frac{\partial}{\partial t} \left(\frac{1}{\mu_{r1}} \underline{B}^a \right) = 0 \qquad (12)$$

$$\underline{\nabla} \times \left(\frac{1}{\mu_{r2}} \underline{B}^a \right) - \frac{1}{c^2} \frac{\partial}{\partial t} \left(\varepsilon_{r2} \underline{E}^a \right) = 0 \qquad (13)$$

A resonance behaviour can be computed by transforming these equations to a Helmholtz-like equation system as described above. Instead of j and J, now the relative dielectricity and permeability have to be predefined as spatial functions.