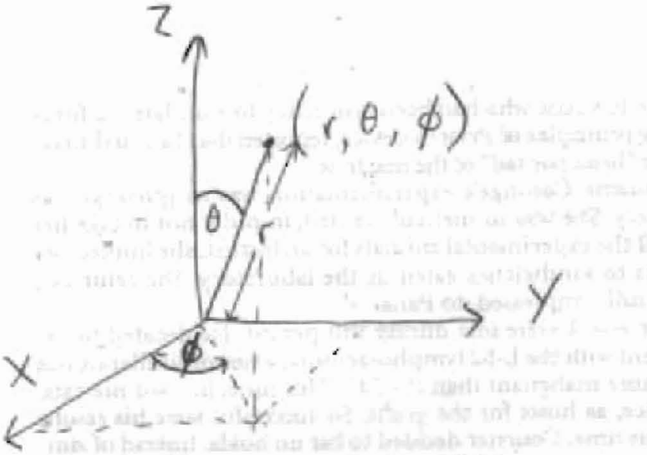
1) Spherical Polar Coordinates



$$\left.\begin{aligned} x &= r\sin\theta\cos\phi \\ y &= r\sin\theta\sin\phi \\ z &= r\cos\theta \end{aligned}\right\} \quad (1)$$

The position vector is

$$\underline{r} = x\,\underline{i} + y\,\underline{j} + z\,\underline{k} \quad (2)$$

where

$$\underline{i} = \sin\theta\cos\phi\,\underline{e}_r + \cos\theta\cos\phi\,\underline{e}_\theta - \sin\phi\,\underline{e}_\phi$$

$$\underline{j} = \sin\theta\sin\phi\,\underline{e}_r + \cos\theta\sin\phi\,\underline{e}_\theta + \cos\phi\,\underline{e}_\phi$$

$$\underline{k} = \cos\theta\,\underline{e}_r - \sin\theta\,\underline{e}_\theta$$

$$\underline{e}_r = \sin\theta\cos\phi\,\underline{i} + \sin\theta\sin\phi\,\underline{j} + \cos\theta\,\underline{k}$$

$$\underline{e}_\phi = -\sin\phi\,\underline{i} + \cos\phi\,\underline{j}$$

$$\underline{e}_\theta = \cos\theta\cos\phi\,\underline{i} + \cos\theta\sin\phi\,\underline{j} - \sin\theta\,\underline{k}$$

We also have:

$$\dot{\underline{e}}_r = \dot\theta\,\underline{e}_\theta + \sin\theta\,\dot\phi\,\underline{e}_\theta$$

$$\dot{\underline{e}}_\theta = -\dot\theta\,\underline{e}_r + \cos\theta\,\dot\phi\,\underline{e}_\theta$$

$$\dot{\underline{e}}_\phi = -\sin\theta\,\dot\phi\,\underline{e}_r - \cos\theta\,\dot\phi\,\underline{e}_\theta$$

The velocity of a particle in spherical polar coordinates is:

a)

$$\underline{V} = V_r \underline{e}_r + V_\theta \underline{e}_\theta + V_\phi \underline{e}_\phi \quad -(3)$$

$$= \dot{r} \underline{e}_r + r\dot{\theta} \underline{e}_\theta + r\sin\theta\,\dot{\phi}\, \underline{e}_\phi$$

The acceleration of a particle is:

$$\underline{a} = a_r \underline{e}_r + a_\theta \underline{e}_\theta + a_\phi \underline{e}_\phi$$

here: $a_r = \ddot{r} - r\dot{\theta}^2 - r\sin^2\theta\,\dot{\phi}^2$

$$a_\theta = 2\dot{r}\dot{\theta} + r\ddot{\theta} - r\sin\theta\cos\theta\,\dot{\phi}^2$$

$$a_\phi = 2\dot{r}\dot{\phi}\sin\theta + 2r\dot{\theta}\dot{\phi}\cos\theta + r\sin\theta\,\ddot{\phi}$$

The magnetic field is:

$$\underline{B} = B_r \underline{e}_r + B_\theta \underline{e}_\theta + B_\phi \underline{e}_\phi \quad -(4)$$

Z Axis Magnetic Field

$$\underline{B} = B_Z \underline{k} \quad -(5)$$

$$= B_Z\cos\theta\, \underline{e}_r - B_Z\sin\theta\, \underline{e}_\theta$$

X Axis Velocity

$$\underline{V} = V_X \underline{i}$$

$$= V_X\sin\theta\cos\phi\, \underline{e}_r + V_X\cos\theta\cos\phi\, \underline{e}_\theta - V_X\sin\phi\, \underline{e}_\phi \quad -(6)$$

3) If $\underline{v}$ is in x and $\underline{B}$ is in z the cross product

is
$$\underline{v}\times\underline{B} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ v_x & 0 & 0 \\ 0 & 0 & B_z \end{vmatrix} = -v_x B_z \underline{j}$$

$$= -v_x B_z\left(\sin\theta\cos\phi\,\underline{e}_r + \cos\theta\cos\phi\,\underline{e}_\theta - \sin\phi\,\underline{e}_\phi\right)$$

General Curl

If: $\underline{A} = A_r\underline{e}_r + A_\theta\underline{e}_\theta + A_\phi\underline{e}_\phi$

$\underline{B} = B_r\underline{e}_r + B_\theta\underline{e}_\theta + B_\phi\underline{e}_\phi$

then:
$$\underline{A}\times\underline{B} = \begin{vmatrix} \underline{e}_r & \underline{e}_\theta & \underline{e}_\phi \\ A_r & A_\theta & A_\phi \\ B_r & B_\theta & B_\phi \end{vmatrix} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

We have:
$$\underline{e}_r\times\underline{e}_\theta = \left(\sin\theta\cos\phi\,\underline{i} + \sin\theta\sin\phi\,\underline{j} + \cos\theta\,\underline{k}\right)\times\left(\cos\theta\cos\phi\,\underline{i} + \cos\theta\sin\phi\,\underline{j} - \sin\theta\,\underline{k}\right)$$

$$= -\left(\sin^2\theta\sin\phi + \cos^2\theta\sin\phi\right)\underline{i} + \cos\phi\left(\sin^2\theta + \cos^2\theta\right)\underline{j}$$

$$= -\sin\phi\,\underline{i} + \cos\phi\,\underline{j} = \underline{e}_\phi$$

$$\boxed{\begin{array}{ll} \underline{e}_r\times\underline{e}_\theta = \underline{e}_\phi & \underline{e}_r\cdot\underline{e}_\theta = 0 \\ \underline{e}_\theta\times\underline{e}_\phi = \underline{e}_r & \underline{e}_\theta\cdot\underline{e}_\phi = 0 \\ \underline{e}_\phi\times\underline{e}_r = \underline{e}_\theta & \underline{e}_\phi\cdot\underline{e}_r = 0 \end{array}}$$