97(3): Proposed Method of Determining $T_{\mu\nu}$ for the Most General Line Element.

The most general line element is:

$$ds^2 = A(dx_0)^2 - B(dx_1)^2 - C(dx_2)^2 - D(dx_3)^2 \quad -(1)$$

in orthogonal curvilinear coordinates. The method proposed is to use Maxima to evaluate the non-zero Christoffel symbols and Riemann tensor elements from (1), test for the first Bianchi identity:

$$R \wedge q = 0 \quad -(2)$$

and then evaluate the left hand side of

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = k T_{\mu\nu} \quad -(3)$$

in terms of A, B, C and D. Eq (3) is the Einstein - Hilbert field equation where $R_{\mu\nu}$ is the Ricci tensor, R is the Ricci scalar curvature, $g_{\mu\nu}$ is the metric and $T_{\mu\nu}$ is the canonical energy (density momentum) tensor of Noether. In eq (3) k is Einstein's constant. Thus $T_{\mu\nu}$ is expressed in terms of A, B, C and D for the system of coordinates being used. To determine A, B, C and D the system of simultaneous equations is solved numerically for given initial and boundary conditions. This gives the $T_{\mu\nu}$ tensor from the line - element (1)

2)

A particular model is introduced for $T_{\mu\nu}$. It can then be deduced what A, B, C and D are needed in the line element (1) for this model. Examples are one electron oscillating in the Z axis of the Cartesian system of coordinates, or one electron rotating around the Z axis. The computer can also evaluate the radius of curvature and geodesic proper radius, and show that they are different in general. It can be checked that all the requirements of differential geometry are fulfilled, and that boundary conditions are satisfied.

<u>Models for $T_{\mu\nu}$</u>

This tensor determines the curvature of space and its properties are described in L. H. Ryder, "Quantum Field Theory" (CUP, 2nd ed., 1996), pp. 88 ff. The angular momentum density for example is:

$$M^{0\mu\nu} = T^{0\mu}x^{\nu} - T^{0\nu}x^{\mu} \quad - (4)$$

and the angular momentum is:

$$M^{\mu\nu} = \int M^{0\mu\nu}\, d^3x . \quad - (5)$$

The Noether Theorem is:

$$D_{\mu} T^{\mu\nu} = 0 \quad - (6)$$

As described by Ryder on his page 90 the three components of the angular momentum are M^{12}

3) M^{23} and M^{31}. These can be described as spin angular momenta (see paper 55 of www.aias.us). Then M^{01}, M^{02} and M^{03} are related to the centre of mass of the system and can be described as orbital angular momenta. The angular momentum tensor is conserved

$$\frac{d}{dt} M^{\mu\nu} = 0 \qquad -(7)$$

and in a curved spacetime:

$$D_\rho M^{\rho\mu\nu} = 0 \qquad -(8)$$

where:

$$M^{\rho\mu\nu} = T^{\rho\mu} x^\nu - T^{\rho\nu} x^\mu \qquad -(9)$$

Thus:

$$\left(D_\rho T^{\rho\mu}\right) x^\nu + T^{\rho\mu} D_\rho x^\nu - \left(D_\rho T^{\rho\nu}\right) x^\mu - T^{\rho\nu} D_\rho x^\mu = 0 \qquad -(10)$$

Using the Noether theorem:

$$T^{\rho\mu} D_\rho x^\nu = T^{\rho\nu} D_\rho x^\mu \qquad -(11)$$

The LHS is true when $\rho = \nu$ and the RHS is true when $\rho = \mu$, so:

$$T^{\nu\mu} = T^{\mu\nu} \qquad -(12)$$

Eq (6) is conservation of energy / momentum density and eq (7) is conservation of angular momentum.

4)

The orbital angular momentum in classical dynamics is

$$\underline{J} = \underline{p} \times \underline{r} \quad - (13)$$

So it is seen by comparison of eqns. (4) and (13) that:

$$m^{012} = T^{01} x^2 - T^{02} x^1 \quad - (14)$$

$$m^{12} = \int m^{012} \, d^3 x \quad - (15)$$

etc. are tensor representations of eq. (13). We have:

$$x^\mu = (ct, X, Y, Z) \quad - (16)$$
$$= (x^0, x^1, x^2, x^3)$$

$$p^\mu = \left(\frac{En}{c}, p_X, p_Y, p_Z \right) \quad - (17)$$
$$= (p^0, p^1, p^2, p^3)$$

So:

$$\frac{p^0}{V} = T^{00} = \frac{En}{Vc} \quad - (18)$$

$$\frac{p^1}{V} = T^{01} = \frac{p_X}{V} \quad - (19)$$

$$\frac{p^2}{V} = T^{02} = \frac{p_Y}{V} \quad - (20)$$

$$\frac{p^3}{V} = T^{03} = \frac{p_Z}{V} \quad - (21)$$

5)

Radiating Electron

This must obey the equations:

$$\underline{\nabla} \cdot \underline{B} = 0 \quad - (22)$$

$$\underline{\nabla} \times \underline{E} + \frac{\partial \underline{B}}{\partial t} = \underline{0} \quad - (23)$$

$$\underline{\nabla} \cdot \underline{E} = \rho / \epsilon_0 \quad - (24)$$

$$\underline{\nabla} \times \underline{B} - \frac{1}{c^2} \frac{\partial \underline{E}}{\partial t} = \mu_0 \underline{J} \quad - (25)$$

so $\rho(\underline{r}, t)$ and $\underline{J}(r, t)$ must be function of r & t. The charge density $\rho(\underline{r}, t)$ and current density $\underline{J}(r, t)$ are furthermore found from the four components of $T^{\mu\nu}$ in eqs. (18) to (21).

So the system of eqns. (3) reduce to:

$$R_{00} - \frac{1}{2} R g_{00} = k T_{00}(\underline{r}, t) \quad - (26)$$

$$R_{11} - \frac{1}{2} R g_{11} = k T_{11}(\underline{r}, t) \quad - (27)$$

$$R_{22} - \frac{1}{2} R g_{22} = k T_{22}(\underline{r}, t) \quad - (28)$$

$$R_{33} - \frac{1}{2} R g_{33} = k T_{33}(\underline{r}, t) \quad - (29)$$

with:

$$T_{00}(\underline{r}, t) = \rho_0(\underline{r}, t) / \bar{V} \quad - (30)$$

$$T_{01}(\underline{r}, t) = \rho_1(\underline{r}, t) / \bar{V} \quad - (31)$$

$$T_{02}(\underline{r}, t) = \rho_2(\underline{r}, t) / \bar{V} \quad - (32)$$

$$T_{03}(\underline{r}, t) = \rho_3(\underline{r}, t) / \bar{V} \quad - (33)$$

Therefore eqns. (26) to (29) define A, B, C and D in eq. (1) in a particular coordinate system in terms of $p_0(\underline{r}, t)$, $p_1(\underline{r}, t)$, $p_2(\underline{r}, t)$ and $p_3(\underline{r}, t)$. Using the minimal prescription:

$$p_\mu = e A_\mu \quad -(34)$$

we have defined A, B, C and D in terms of the four potential A_μ of ECE theory, and thus in terms of the electric and magnetic fields of a radiating electron and the spin connection:

$$\underline{B} = \underline{\nabla} \times \underline{A} - \underline{\omega} \times \underline{A} \quad -(35)$$

$$\underline{E} = -\underline{\nabla} \phi - \frac{\partial \underline{A}}{\partial t} - c\omega^0 \underline{A} + c\underline{\omega}\phi \quad -(36)$$

Here:

$$A^\mu = (c\phi, \underline{A}). \quad -(37)$$

The system of equations (26) to (29) is:

$$\boxed{\begin{aligned} R_{00} - \tfrac{1}{2} R g_{00} &= ek A_0(\underline{r}, t) / V \\ R_{11} - \tfrac{1}{2} R g_{11} &= ek A_1(\underline{r}, t) / V \\ R_{22} - \tfrac{1}{2} R g_{22} &= ek A_2(\underline{r}, t) / V \\ R_{33} - \tfrac{1}{2} R g_{33} &= ek A_3(\underline{r}, t) / V \end{aligned}} \quad \begin{aligned} &-(38) \\ &-(39) \\ &-(40) \\ &-(41) \end{aligned}$$

where

$$A_0 = c\phi \quad -(42)$$

where A^μ must obey eqns. (22) - (25), (35) and (36). Using computer algebra, eqns. (22) - (25) can be expressed in terms of A^μ and ω^μ, where:

$$\omega^\mu = (\omega^0 , \underline{\omega}) . \quad - (43)$$

The charge - current density is $R^{\kappa\nu}$ multiplied by $-A^{(0)}$:

$$J^{\kappa\nu} = - A^{(0)} R^{\kappa}{}_{\mu}{}^{\mu\nu} . \quad - (44)$$

So these equations define a radiating electron in ECE theory. They must be solved self-consistently for any given method.

Vacuum Solution

This is:

$$R_{00} - \frac{1}{2} R g_{00} = 0 \quad - (45)$$

$$R_{11} - \frac{1}{2} R g_{11} = 0 \quad - (46)$$

$$R_{22} - \frac{1}{2} R g_{22} = 0 \quad - (47)$$

$$R_{33} - \frac{1}{2} R g_{33} = 0 \quad - (48)$$

In this case A, B, C and D of eq. (1) are constrained by eqs. (45) to (48).