94(b): Electric Field in Vector Notation

For the electric field we consider:

$$F^1{}_{01} = (\partial_0 A_1 - \partial_1 A_0)^1 + \omega^1{}_{01} A^1{}_1 - \omega^1{}_{10} A^0{}_0 \quad -(1)$$

$$F^2{}_{02} = (\partial_0 A_2 - \partial_2 A_0)^2 + \omega^2{}_{02} A^2{}_2 - \omega^2{}_{20} A^0{}_0 \quad -(2)$$

$$F^3{}_{03} = (\partial_0 A_3 - \partial_3 A_0)^3 + \omega^3{}_{03} A^3{}_3 - \omega^3{}_{30} A^0{}_0 \quad -(3)$$

In the vector notation of previous papers:

$$\underline{E} = -\underline{\nabla}\phi - \frac{\partial \underline{A}}{\partial t} - c\omega^0 \underline{A} + c\underline{\omega}\phi \quad -(4)$$

Therefore:

$$-\left(\underline{\nabla}\phi + \frac{\partial \underline{A}}{\partial t}\right)_x = (\partial_0 A_1 - \partial_1 A_0)^1 \quad -(5)$$

$$-\left(\underline{\nabla}\phi + \frac{\partial \underline{A}}{\partial t}\right)_y = (\partial_0 A_2 - \partial_2 A_0)^2 \quad -(6)$$

$$-\left(\underline{\nabla}\phi + \frac{\partial \underline{A}}{\partial t}\right)_Z = (\partial_0 A_3 - \partial_3 A_0)^3 \quad -(7)$$

and:

$$-\left(c\omega^0 \underline{A} - c\underline{\omega}\phi\right)_x = \omega^1{}_{01} A^1{}_1 - \omega^1{}_{10} A^0{}_0 \quad -(8)$$

$$-\left(c\omega^0 \underline{A} - c\underline{\omega}\phi\right)_y = \omega^2{}_{02} A^2{}_2 - \omega^2{}_{20} A^0{}_0 \quad -(9)$$

$$-\left(c\omega^0 \underline{A} - c\underline{\omega}\phi\right)_Z = \omega^3{}_{03} A^3{}_3 - \omega^3{}_{30} A^0{}_0 \quad -(10)$$

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Thus:

$$\underline{A} = A_X \underline{i} + A_Y \underline{j} + A_Z \underline{k} \quad -(11)$$

where:

$$A_X = A^1{}_1, \quad A_Y = A^2{}_2, \quad A_Z = A^3{}_3, \quad -(12)$$

and

$$\underline{\omega} = \omega_X \underline{i} + \omega_Y \underline{j} + \omega_Z \underline{k} \quad -(13)$$

where:

$$\boxed{\omega_X = \omega^1{}_{10}, \quad \omega_Y = \omega^2{}_{20}, \quad \omega_Z = \omega^3{}_{30}} \quad -(14)$$

The scalar part of the spin connection is defined by:

$$\boxed{\omega^0 = -\frac{\omega^1{}_{01}}{c} = -\frac{\omega^2{}_{02}}{c} = -\frac{\omega^3{}_{03}}{c}} \quad -(15)$$

and the scalar potential is defined by:

$$\boxed{\phi = -\frac{A^0{}_0}{c}} \quad -(16)$$

Final Result

$$\boxed{\begin{aligned} \underline{E} &= -\underline{\nabla}\phi - \frac{\partial \underline{A}}{\partial t} - c\omega^0 \underline{A} + c\underline{\omega}\phi \\ \underline{B} &= \underline{\nabla} \times \underline{A} - \underline{\omega} \times \underline{A} \end{aligned}}$$

As in paper 94 and previous papers