1)

93(9a): Calculation of Riemann Elements, Part 2

$$R^1{}_{212} = \partial_1 \Gamma^1_{22} - \partial_2 \Gamma^1_{12} + \Gamma^1_{1\lambda}\Gamma^\lambda_{22} - \Gamma^1_{2\lambda}\Gamma^\lambda_{12}$$

$$= \partial_1 \Gamma^1_{22} + \Gamma^1_{11}\Gamma^1_{22} - \Gamma^1_{22}\Gamma^2_{12}$$

$$= -\partial_1\left(re^{-2\beta}\right) - r(\partial_1\beta)e^{-2\beta} + \frac{r}{r}e^{-2\beta}$$

$$\boxed{R^1{}_{212} = re^{-2\beta}\partial_1\beta} \quad \checkmark\checkmark$$

$$R^1{}_{313} = \partial_1\Gamma^1_{33} + \Gamma^1_{11}\Gamma^1_{33} - \Gamma^1_{33}\Gamma^3_{13}$$

$$= -\partial_1\left(re^{-2\beta}\sin^2\theta\right) - re^{-2\beta}\sin^2\theta(\partial_1\beta) + \frac{r}{r}e^{-2\beta}\sin^2\theta$$

$$\boxed{R^1{}_{313} = re^{-2\beta}\sin^2\theta\,\partial_1\beta} \quad \checkmark\checkmark$$

$$R^2{}_{323} = \partial_2\Gamma^2_{33} - \partial_3\Gamma^2_{23} + \Gamma^2_{2\lambda}\Gamma^\lambda_{33} - \Gamma^2_{3\lambda}\Gamma^\lambda_{23}$$

$$= \partial_2\Gamma^2_{33} + \Gamma^2_{21}\Gamma^1_{33} - \Gamma^2_{33}\Gamma^3_{23}$$

$$= -\frac{\partial}{\partial\theta}\left(\sin\theta\cos\theta\right) - \frac{r}{r}e^{-2\beta}\sin^2\theta + \sin\theta\cos\theta\,\frac{\cos\theta}{\sin\theta}$$

$$= -\cos^2\theta + \sin^2\theta - e^{-2\beta}\sin^2\theta + \cos^2\theta$$

$$\boxed{R^2{}_{323} = \sin^2\theta\left(1 - e^{-2\beta}\right)} \quad \checkmark\checkmark$$