APPENDIX 3 : REDUCTION TO VECTOR NOTATION

In this appendix the tensorial form of the inhomogeneous ECE equation is reduced to the vector form, giving the Coulomb and Ampère Maxwell laws in generally covariant unified field theory. Begin with the inhomogeneous field equation:

$$\partial_\mu F^{a\mu\nu} = \mu_0 J^{a\nu} = -\frac{A^{(0)}}{\mu_0}\left(R^{a\ \ \mu\nu}_{\ \mu} + \omega^{a}_{\ \mu b} T^{b\mu\nu}\right). \quad -(C1)$$

In the Einstein Hilbert limit:

$$T^{b\mu 0} = 0 \quad -(C2)$$

so the equation becomes:

$$\partial_\mu F^{a\mu\nu} = -\frac{A^{(0)}}{\mu_0} R^{a\ \ \mu\nu}_{\ \mu}. \quad -(C3)$$

The indices in the Riemann tensor elements are raised using the metric of the base manifold as follows:

$$R^{a\ \ \sigma\rho}_{\ \mu} = g^{\sigma\nu} g^{\rho\kappa} R^{a}_{\ \mu\nu\kappa}. \quad -(C4)$$

The Coulomb Law is obtained for:

$$\nu = 0 \quad -(C5)$$

and is:

$$\partial_\mu F^{a\mu 0} = -\frac{A^{(0)}}{\mu_0}\left(R^{a\ \ 10}_{\ 1} + R^{a\ \ 20}_{\ 2} + R^{a\ \ 30}_{\ 3}\right) \quad -(C6)$$

where summation over repeated μ indices has been carried out. The vector form of eq. (C6) is:

$$(\underline{\nabla} \cdot \underline{E})^a = -\phi \left(R^a{}_1{}^{10} + R^a{}_2{}^{20} + R^a{}_3{}^{30} \right). \quad -(C7)$$

The only possible value of a (see also Appendix Four) for the Coulomb Law is:

$$a = 0 \quad -(C8)$$

so we obtain the generally covariant Coulomb Law:

$$\underline{\nabla} \cdot \underline{E} = (\underline{\nabla} \cdot \underline{E})^0 = -\phi \left(R^0{}_1{}^{10} + R^0{}_2{}^{20} + R^0{}_3{}^{30} \right). \quad -(C9)$$

Both sides are scalar valued quantities, so the time-like, or scalar, index a = 0 is used. Here ϕ is the scalar potential, having the units of volts. The units of $\underline{E}$ are volt / m and those of the R elements are inverse meters squared, so units are consistent.

The generally covariant Ampère Maxwell law is obtained with:

$$\nu = 1, 2, 3. \quad -(C10)$$

When:

$$\nu = 1 \quad -(C11)$$

Eq. (C3) becomes:

$$\partial_0 F^{a01} + \partial_2 F^{a21} + \partial_3 F^{a31} = -\frac{A^{(0)}}{\mu_0} R^a{}_\mu{}^{\mu 1}. \quad -(C12)$$

The vector form of this equation is:

$$(\underline{\nabla} \times \underline{B})^a_1 = \frac{1}{c^2}\frac{\partial E^a_1}{\partial t} + \frac{\mu_0}{c} J^a_1 . \quad -(C13)$$

Here, the 1 subscript denotes a component in a particular coordinate system. For example in the spherical polar system:

$$1 = r \quad -(C14)$$

or in the Cartesian system:

$$1 = X . \quad -(C15)$$

So Eq. (C13) is the r or X component of the Ampère Maxwell Law. If we adopt the spherical polar system for the Riemann elements (see Appendix 5) the value of a in Eq. (C13) must also be 1. If the complex circular basis $\{\partial - a\}$ is chosen then:

$$a = (1), (2), (3) . \quad -(C16)$$

However, if the complex circular basis is chosen, then the relevant Riemann elements are:

$$R^{(1)\ \ \mu 1}_{\ \ \ \ \mu} , \ R^{(2)\ \ \mu 2}_{\ \ \ \ \mu} , \ R^{(3)\ \ \mu 3}_{\ \ \ \ \mu} \quad -(C17)$$

in which one index is complex circular, and the other three are spherical polar. It is possible to use either system, or any other system of coordinates for a. Therefore the generally covariant Ampère Maxwell Law is:

$$\underline{\nabla} \times \underline{B} = \frac{1}{c^2}\frac{\partial \underline{E}}{\partial t} + \frac{\mu_0}{c} \underline{J} \quad -(C18)$$

where the charge current density is defined as:

$$\underline{J} = J^1_1 \underline{e}_r + J^2_2 \underline{e}_\theta + J^3_3 \underline{e}_\phi \quad -(C19)$$

with the scalar valued components:

$$J^1_1 = -\frac{A^{(0)}}{\mu_0}\left(R^1{}_0{}^{01} + R^1{}_2{}^{21} + R^1{}_3{}^{31}\right), \quad -(C20)$$

$$J^2_2 = -\frac{A^{(0)}}{\mu_0}\left(R^2{}_0{}^{02} + R^2{}_1{}^{12} + R^2{}_3{}^{32}\right), \quad -(C21)$$

$$J^3_3 = -\frac{A^{(0)}}{\mu_0}\left(R^3{}_0{}^{03} + R^3{}_1{}^{13} + R^3{}_2{}^{23}\right). \quad -(C22)$$

A particular metric may finally be used to calculate these Riemann components exactly, and example is given in detail in Appendix 5.

The main result is that in the presence of space-time curvature, the electro-dynamical properties of light are changed, in addition to the well known effects of Einstein Hilbert theory there are polarization changes in light deflected by gravitation. These are due to the charge current density $\underline{J}$, which does not exist in the free space limit of Maxwell Heaviside theory. So these are predictions of ECE theory that are known already to be corroborated qualitatively {2-9}, because of observations of polarization changes in light deflected by a white dwarf for example.