(1): Development of Spin Connection Resonance

Consider the basic spin connection resonance of page 3. In radial coordinates the basic equation is:

$$\frac{d^2\phi}{dr^2} + \left(\frac{2}{r} + \omega_r\right)\frac{d\phi}{dr} + \frac{\phi}{r^2}\left(2r\omega_r + r^2\frac{d\omega_r}{dr}\right) = -\frac{\rho}{\epsilon_0} \quad — (1)$$

Compare this with the basic resonance equation, which can be found in any textbook such as Marion and Thornton:

$$\ddot{x} + 2\beta\dot{x} + \omega_0^2 x = A\cos\omega t \quad —(2)$$

Now make this into an undamped resonance equation so that

$$\beta = 0, \quad —(3)$$

to obtain:

$$\ddot{x} + \omega_0^2 x = A\cos\omega t \quad —(3)$$

Comparing eqns. (1) and (2)

$$\frac{2}{r} + \omega_r = 0 \quad —(4)$$

$$\frac{2}{r}\omega_r + \frac{d\omega_r}{dr} = \omega_0^2 \quad —(5)$$

i.e.

$$\omega_r = -\frac{2}{r}, \quad \frac{d\omega_r}{dr} = \omega_0^2 - \frac{4}{r^2}$$

$$= \frac{4}{r^2} \quad —(6$$

2)

So $\omega_0^2 = \frac{8}{r^2}$. — (7)

If we consider $\beta \neq 0$, ~~the~~:

$$\frac{2}{r} + \omega_r = 2\beta \quad — (8)$$

$$\frac{2}{r}\omega_r + \frac{\partial \omega_r}{\partial r} = \omega_0^2 \quad — (9)$$

and

$$\frac{4}{r}\left(\beta - \frac{1}{r}\right) + \frac{\partial \omega_r}{\partial r} = \omega_0^2. \quad — (10)$$

Thus:

$$\omega_r = \int \omega_0^2 - \frac{4\beta}{r} + \frac{4}{r^2}\, dr \quad — (12)$$

$$\boxed{\omega_r = \omega_0^2 r - 4\beta \log_e r - \frac{4}{r}} \quad — (13)$$

With this type of spin connection eq. (1) is:

$$\boxed{\frac{\partial^2 \phi}{\partial r^2} + 2\beta \frac{\partial \phi}{\partial r} + \omega_0^2 \phi = -\frac{\rho}{\epsilon_0}} \quad — (14)$$

which is a damped resonator, Q.E.D.

In the case $\omega_0 = 0$, $\beta = 0$, we obtain

$$\omega_r = -\frac{4}{r} \quad — (15)$$