88(3): Estimate of Photon Mass from the Lamb Shift.

The method used is to assume that the radiative correction is due to the ubiquitous background radiation at 2.7 K. We first calculate the energy density of a black body at 2.7 K using the Planck distribution (Atkins, second edition, p. 8). This is:

$$U = \int_0^\infty \frac{8\pi h}{c^3}\left(\frac{\nu^3 \, d\nu}{e^{h\nu/(kT)} - 1}\right)$$

$$= \left(\frac{\pi^2}{15} \cdot \frac{k^4}{c^3 \hbar^3}\right) T^4 \qquad -(1)$$

$$= 4.02 \times 10^{-14} \text{ J m}^{-3} \text{ at } 2.7\text{K}.$$

The energy in a volume V is:

$$E_n = UV. \qquad -(2)$$

Here V is <u>a volume of radiation</u>.

It is known experimentally that the Lamb shift in atomic H (2s − 2p) is:

$$E_L = 0.0353 \text{ cm}^{-1} \qquad -(3)$$

$$= 3.53 \text{ m}^{-1} \qquad -(4)$$

$$= 3.53 \, \hbar c \text{ joules} \qquad -(5)$$

2) Here:

$$\hbar = 1.0546 \times 10^{-34} \text{ Js}$$
$$c = 2.998 \times 10^{8} \text{ m s}^{-1}$$

So:

$$E_L(\text{Lamb}) = 1.161 \times 10^{-25} \text{ joules},$$

and

$$U(\text{Lamb}) = \frac{1.161}{V} \times 10^{-25} \text{ J m}^{-3} \quad \text{— (6)}$$

where V is a volume of radiation.

We now use the low frequency limit of the Planck distribution: — (7)

$$dU = \epsilon \, dN = \frac{8\pi \hbar \nu^3}{c^3} \left(\frac{e^{-\hbar\nu/kT}}{1 - e^{-\hbar\nu/kT}} \right) d\nu$$

where:

$$\omega = 2\pi\nu \quad \text{— (8)}$$

3)

$$\lim_{\nu \to 0} dU = \frac{8\pi \nu^2 kT}{c^3} d\nu \qquad (9)$$

So:

$$\boxed{U = \frac{8\pi}{3} \frac{\nu^3}{c^3} kT} \qquad (10)$$

where:

$$\omega = 2\pi\nu. \qquad (11)$$

As $\omega \to 0$, it is known from ~~the~~ de Broglie theorem that:

$$\hbar\omega = mc^2 \qquad (12)$$

where m is the mass of the photon. So:

$$\omega = \frac{mc^2}{\hbar}, \quad \boxed{\nu = \frac{mc^2}{2\pi\hbar}} \qquad (13)$$

From eqs. (8) and (11):

$$U = \frac{8\pi}{3} \frac{m^3 c^6}{8\pi^3 \hbar^3 c^3} kT \qquad (14)$$

$$\boxed{U = \frac{m^3 c^3 kT}{3\pi^2 \hbar^3}} \qquad (15)$$

4)

So the photon mass is:

$$\boxed{m = \left(\frac{3\pi^2 U}{kT}\right)^{1/3} \cdot \frac{\hbar}{c}} \quad - (16)$$

In order to calculate U, the volume of radiation associated with En (Lamb) is needed. We now use the fact that at 2.7K there are 4.0×10^8 photons in one cubic metre of radiation (Atkins). So the energy per photon at 2.7 K is 1.005×10^{-22} Joules. This quantity is the same for any volume, so we may choose:

$$V = 1.0 \, m^3. \quad - (17)$$

Therefore in eq. (16):

$$\left.\begin{aligned} U &= 1.161 \times 10^{-25} \ J m^{-3}, \\ T &= 2.7K. \end{aligned}\right\} - (18)$$

This gives:

$$\boxed{m = 1.59 \times 10^{-44} \ kgm}.$$

This compares with

$$m \lesssim 10^{-39} \ kgm$$

from the Michelson Morley experiment (Lehnert review). The strongest upper limit in the standard model is 1.8×10^{-50} kgm.