8(1): Reduction of ECE to the Lehnert Theory

The equations of the Lehnert theory are as follows:

$$\underline{\nabla} \cdot \underline{B} = 0 \quad -(1)$$

$$\underline{\nabla} \times \underline{E} + \frac{\partial \underline{B}}{\partial t} = \underline{0} \quad -(2)$$

$$\underline{\nabla} \cdot \underline{E} = \rho(\text{vac}) / \epsilon_0 \quad -(3)$$

$$\underline{\nabla} \times \underline{B} - \frac{1}{c^2} \frac{\partial \underline{E}}{\partial t} = \underline{j}(\text{vac}) \quad -(4)$$

$$\underline{B} = \underline{\nabla} \times \underline{A}, \quad \underline{E} = -\underline{\nabla} \phi - \frac{\partial \underline{A}}{\partial t}. \quad -(5)$$

In the Maxwell Heaviside theory:

$$\rho(\text{vac}) = 0, \quad \underline{j}(\text{vac}) = \underline{0}. \quad -(6)$$

Lehnert then deduces his equivalent of the $\underline{B}^{(3)}$ field from these equations, and relates them to photon mass (B. Lehnert, Adv. Chem. Phys., 119 (2) (2001)). Photon mass was introduced by Einstein in 1905:

1) A. Einstein, Ann. Phys., 7, 132 (1905); 18, 121 (1917).

and by Schrödinger and Bass:

2) L. Bass and E. Schrödinger, Proc. Roy. Soc., 232A, 1 (1955).

2) The vacuum charge-current density is:

$$j^\mu(\text{vac}) = \left(\rho(\text{vac})c, \underline{J}(\text{vac})\right) \quad -(7)$$

and exists in the absence of electron mass, i.e. in the absence of a source term.

<u>Reduction of ECE to Lehnert</u>

The ECE field equations are:

$$F^a = d \wedge A^a + \omega^a{}_b \wedge A^b \quad -(8)$$

$$d \wedge F^a = \mu_0 j^a \quad -(9)$$

$$d \wedge \tilde{F}^a = \mu_0 \tilde{j}^a \quad -(10)$$

where:

$$j^a = \frac{A^{(0)}}{\mu_0}\left(R^a{}_b \wedge q^b - \omega^a{}_b \wedge T^b\right) \quad -(11)$$

$$\tilde{j}^a = \frac{A^{(0)}}{\mu_0}\left(\tilde{R}^a{}_b \wedge q^b - \omega^a{}_b \wedge \tilde{T}^b\right) \quad -(12)$$

in the standard notation of Cartan geometry.

The Lehnert theory in this notation is:

$$d \wedge F = 0 \quad -(13)$$

$$d \wedge \tilde{F} = \mu_0 \tilde{j}(\text{vac}) \quad -(14)$$

$$F = d \wedge A \quad -(15)$$

The Maxwell Heaviside ~~theory~~ is, in the vacuum:

$$d \wedge F = 0 \quad -(16)$$

$$d \wedge \tilde{F} = 0 \quad -(17)$$

$$F = d \wedge A. \quad -(18)$$

The ECE wave equation of electrodynamics is:

$$\left(\Box + kT \right) A^a_\mu = 0 \quad -(19)$$

and reduces to:

$$\left(\Box + \left(\frac{mc}{\hbar} \right)^2 \right) A^a_\mu = 0 \quad -(20)$$

when the electromagnetic field becomes independent of all other fields.

The $\underline{B}^{(3)}$ field is given by the second term of eq. (8) and the photon mass m by eq. (20). The fundamental ECE postulate is:

$$A^a_\mu = A^{(0)} q^a_\mu. \quad -(21)$$

4) It is seen that the Lehnert and MH equations do not use a spin connection $\omega^a{}_b$, so are ~~theories~~ of special relativity, i.e. are Lorentz covariant but not generally covariant as required by general relativity. The ECE ~~theory~~ is a generally covariant unified field theory. Therefore the ECE theory reduces to the Lehnert ~~theory~~ if:

$$\omega^a{}_b = 0 \quad -(22)$$

$$j^a = 0 \quad -(23)$$

$$\tilde{j}^a \neq 0 \quad -(24)$$

and if the existence of the indices a is implied in the Lehnert theory:

This means that in the Lehnert theory:

$$R^a{}_b \wedge q^b = \omega^a{}_b \wedge T^b \quad -(25)$$

i.e. there is no interaction between rotation and translation, the rotation is pure rotation. Lehnert assumes that in the absence of electron mass:

$$\tilde{R}^a{}_b \wedge q^b \neq \omega^a{}_b \wedge \tilde{T}^b \quad -(26)$$

5)

One solution of eq. (25) is:

$$\omega^a{}_b = -\frac{1}{2}\kappa\,\epsilon^a{}_{bc}\,q^c, \quad -(27)$$

$$R^a{}_b = -\frac{1}{2}\kappa\,\epsilon^a{}_{bc}\,T^c. \quad -(28)$$

This solution means that $\omega^a{}_b$ is the dual of q^c in Minkowski spacetime, (in the base manifold) and $R^a{}_b$ is the dual of T^c. The Hodge duals) of T^c and $R^a{}_b$ are denoted $\tilde{T}^c$ and $\tilde{R}^a{}_b$. In four dimensions these Hodge duals are anti-symmetric tensors whose indices a, b and c are however unchanged. So:

$$\tilde{R}^a{}_b = -\frac{1}{2}\kappa\,\epsilon^a{}_{bc}\,\tilde{T}^c \quad -(29)$$

and

$$\tilde{R}^a{}_b \wedge q^b = \omega^a{}_b \wedge \tilde{T}^b. \quad -(30)$$

So for pure rotational motion in the absence of [electron mass], the ECE theory states that:

$$j^a = 0 \quad -(31)$$

$$\tilde{j}^a = 0 \quad -(32)$$

and

$$d \wedge F^a = 0 \quad -(33)$$

$$d \wedge \tilde{F}^a = 0 \quad -(34)$$

6) In order to make the Lehnert theory self-consistent it is necessary that:

$$j^a = \frac{A^{(0)}}{\mu_0}\left(R^a{}_b \wedge q^b - \omega^a{}_b \wedge T^b\right) \quad -(35)$$

$$\neq 0$$

and

$$\tilde{j}^a = \frac{A^{(0)}}{\mu_0}\left(\tilde{R}^a{}_b \wedge q^b - \omega^a{}_b \wedge \tilde{T}^b\right) \quad -(36)$$

$$\neq 0$$

in the absence of a source term, i.e. in the absence of an electron. The self-consistent Lehnert equations are therefore:

$$d \wedge F^a = \mu_0 j^a \quad -(37)$$

$$d \wedge \tilde{F}^a = \mu_0 \tilde{j}^a . \quad -(38)$$

The only mass present in the absence of an electron is the photon mass m. Therefore the wave equation is:

$$\left(\Box + \left(\frac{mc}{\hbar}\right)^2\right) A^a_\mu = 0 \quad -(39)$$

for each a, which is the Proca equation in the absence of an electron. Finally the e/m field has to be

7) related to the e/m potential with a spin connection

$$F^a = d \wedge A^a + \omega^a{}_b \wedge A^b \quad -(40)$$

The $\underline{B}^{(3)}$ field originates in the spin connection.

The effect of photon mass is to induce the existence of j^a and $\tilde{j}^a$ in the absence of electron mass. This means that photon mass is inter alia determined by the geometry of eqns. (35) and (36) [and not electron mass]. If there were no photon mass, we would have the Maxwell Heaviside equations for each a, i.e.:

$$d \wedge F^a = 0 \quad -(42)$$

$$d \wedge \tilde{F}^a = 0 \quad -(43)$$

$$\Box A^a = 0. \quad -(44)$$

Conclusion

The Lehnert theory is intermediate between MH and ECE, and is not entirely self-consistent. It is still a Lorentz covariant theory of special relativity and for this reason not yet unified with other fundamental fields.