37(7): The Basic Euler Transform (Paper 63)

The starting equation is:

$$\frac{d^2\phi}{dr^2} + \frac{1}{r}\frac{d\phi}{dr} - \frac{1}{r^2}\phi = -\frac{\rho}{\epsilon_0} \quad -(1)$$

This is transformed into a resonance equation using:

$$\kappa r = \exp(i\kappa R) \quad -(2)$$

Thus:

$$\frac{dR}{dr} = -\frac{i}{\kappa r} \quad -(3)$$

and:

$$\frac{d\phi}{dr} = \frac{d\phi}{dR}\frac{dR}{dr} = -\frac{i}{\kappa r}\frac{d\phi}{dR} \quad -(4)$$

$$\boxed{r\frac{d\phi}{dr} = -\frac{i}{\kappa}\frac{d\phi}{dR}} \quad -(5)$$

The second derivative is:

$$\frac{d^2\phi}{dr^2} = \frac{d}{dr}\left(\frac{d\phi}{dr}\right) = \frac{d}{dr}\left(-\frac{i}{\kappa r}\frac{d\phi}{dR}\right)$$

$$= \frac{i}{\kappa r^2}\frac{d\phi}{dR} - \frac{i}{\kappa r}\frac{d}{dr}\left(\frac{d\phi}{dR}\right). \quad -(6)$$

Now use the commutativity property:

$$\frac{d}{dr}\left(\frac{d\phi}{dR}\right) = \frac{d^2\phi}{dr\,dR} = \frac{d^2\phi}{dR\,dr} = \frac{d}{dR}\left(\frac{d\phi}{dr}\right) \quad -(7)$$

So in eq. (6):

$$\frac{d^2\phi}{dr^2} = \frac{i}{\kappa r^2}\frac{d\phi}{dR} - \frac{i}{\kappa r}\frac{d}{dR}\left(\frac{d\phi}{dr}\right)$$

2)

and

$$\frac{d^2\phi}{dr^2} = \frac{i}{\kappa r^2}\frac{d\phi}{dR} - \frac{1}{\kappa^2 r^2}\frac{d^2\phi}{dR^2}$$

i.e

$$\boxed{r^2\frac{d^2\phi}{dr^2} = \frac{i}{\kappa}\frac{d\phi}{dR} - \frac{1}{\kappa^2}\frac{d^2\phi}{dR^2}} \quad -(8)$$

Eq. (1) is:

$$r^2\frac{d^2\phi}{dr^2} + r\frac{d\phi}{dr} - \phi = -r^2\frac{\rho}{\epsilon_0} \quad -(9)$$

The resonance equation from eqs. (5), (8) and (9) is:

$$\boxed{\frac{d^2\phi}{dR^2} + \kappa^2\phi = \exp(2i\kappa R)\frac{\rho}{\epsilon_0}} \quad -(10)$$

where:

$$\exp(2i\kappa R) = \cos(2\kappa R) + i\sin(2\kappa R) \quad -(11)$$

Eq. (10) is an Euler Bernoulli resonance equation, whose <u>driving term</u> is $\exp(2i\kappa R)\rho/\epsilon_0$. It is an <u>undamped</u> oscillator, so even if the driving term is very small initially, resonance amplification of ϕ to <u>infinity</u> can still occur. <u>We</u> now input ρ <u>from the radiative correction.</u>