86(1). Do Negative Energy Solutions Exist?

In developing Rose notes a standard model text such as L. H. Ryder "Quantum Field Theory" may be used (CUP, 2nd. ed. 1996).
On the foot of page 44 Ryder just asserts that the Dirac equation:

$$(\gamma^\mu \hat{p}_\mu - mc)\phi = 0 \quad -(1)$$

produces:

$$\left.\begin{aligned} E &= (m^2c^2 + p^2)^{1/2} \text{ twice} \\ E &= -(m^2c^2 + p^2)^{1/2} \text{ twice.} \end{aligned}\right\} -(2)$$

However, using Ryder's own definition of γ^μ in his eq. (2.92), it is not possible to obtain eq. (2) from eq. (1). This is considered as follows. First set up eq. (1) for a particle at rest:

$$(\gamma^0 \hat{p}_0 - mc)\phi(E_n) = 0 \quad -(3)$$

where

$$\hat{p}_0 = \frac{\hat{E}_n}{c} \quad -(4)$$

So:

$$\boxed{\gamma^0 \hat{E}_n \phi = mc^2\phi} \quad -(5)$$

where:

$$\hat{E}_n = i\hbar\frac{\partial}{\partial t}. \quad -(6)$$

Now write out equation (5) in full:

2)

$$\begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \left(i\hbar \frac{\partial}{\partial t} \right) \begin{bmatrix} \phi_1^R \\ \phi_2^R \\ \phi_1^L \\ \phi_2^L \end{bmatrix} = mc^2 \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \phi_1^R \\ \phi_2^R \\ \phi_1^L \\ \phi_2^L \end{bmatrix} \quad -(7)$$

i.e.:

$$i\hbar \frac{\partial}{\partial t} \phi_2^L = mc^2 \phi_1^R \quad \longrightarrow (8)$$

$$i\hbar \frac{\partial}{\partial t} \phi_1^L = mc^2 \phi_2^R \quad -(9)$$

$$i\hbar \frac{\partial}{\partial t} \phi_2^R = mc^2 \phi_1^L \quad -(10)$$

$$i\hbar \frac{\partial}{\partial t} \phi_1^R = mc^2 \phi_2^L \quad -(11)$$

So:

$$i\hbar \frac{\partial}{\partial t} \left(\phi_1^L + \phi_2^L \right) = mc^2 \left(\phi_1^R + \phi_2^R \right) \quad -(12)$$

$$i\hbar \frac{\partial}{\partial t} \left(\phi_1^R + \phi_2^R \right) = mc^2 \left(\phi_1^L + \phi_2^L \right) \quad -(13)$$

In deriving eqs (12) and (13) Ryder's own definition have been used.

Now use Ryder's equation following his eq. (2.86) on page 41:

$$\phi^L(0) = \phi^R(0) \quad -(14)$$

for a particle at rest. Thus in eqs. (12) and (13):

3)

$$\psi_1^L + \psi_2^L = \psi_1^R + \psi_2^R \quad -(15)$$

so:

$$\boxed{\begin{aligned}\psi_1^L + \psi_2^L &= \psi_1^R + \psi_2^R \\ &= \exp\left(-\frac{imc^2}{\hbar}t\right)\end{aligned}} \quad -(16)$$

This simple algebra does not lead to Ryder's eqns. (2), using Ryder's own definitions.

It is well known that classically:

$$E_0 = mc^2 \quad -(17)$$

is the rest energy. It is never asserted classically that E_0 is negative. In classical special relativity:

$$\underline{p} = \gamma m \underline{v} \quad -(18)$$

is the relativistic momentum. It follows from eq. (18) that:

$$E^2 = c^2 p^2 + E_0^2. \quad -(19)$$

Ryder just asserts that E from eq. (19) can be negative. This is untrue, because eq. (19) comes from squaring eq. (18), (Marion & Thornton). So the reasoning for negative energy particles is deeply flawed.