1)

The Complete Hydrogen Orbitals (Atkins)

Is Orbital

$$\phi(1s) = R_{10} Y_{00} \quad — (1)$$

$$\phi(2s) = R_{20} Y_{00} \quad — (2)$$

$$\phi(2p_z) = R_{21} Y_{10} \quad — (3)$$

$$\phi(2p_x) = R_{21} Y_{11} \quad — (4)$$

$$\phi(2p_y) = R_{21} Y_{1-1} \quad — (5)$$

Where R_{nl} are the radial functions and where Y_{lm_l} are the spherical harmonics.

The Spherical Harmonics (r, θ, ϕ)

$$Y(0,0) = \frac{1}{2\pi^{1/2}} \quad — (6)$$

$$Y(1,0) = \frac{1}{2}\left(\frac{3}{\pi}\right)^{1/2} \cos\theta \quad — (7)$$

$$Y(1,1) = -\frac{1}{2}\left(\frac{3}{2\pi}\right)^{1/2} \sin\theta \, e^{i\phi} \quad — (8)$$

$$Y(1,-1) = \frac{1}{2}\left(\frac{3}{2\pi}\right)^{1/2} \sin\theta \, e^{-i\phi} \quad — (9)$$

The Radial Functions

n	l	$R_{nl}(r)$
1	0 (1s)	$2(1/a)^{3/2} \exp(-r/a)$
2	0 (2s)	$(1/a)^{3/2}(1/(2\sqrt{2}))(2-r/a)\exp(-r/(2a))$
	1 (2p)	$(1/a)^{3/2}(1/(2\sqrt{6}))\frac{r}{a}\exp(-r/(2a))$

2)

So:

$$\boxed{\psi(1s) = \frac{1}{2\pi^{1/2}} \cdot 2\left(\frac{1}{a}\right)^{3/2} \exp\left(-r/a\right)} \quad -(10)$$

$$\boxed{\psi(2s) = \frac{1}{2\pi^{1/2}} \cdot \frac{1}{2\sqrt{2}} \cdot \left(2 - \frac{r}{a}\right) \exp\left(-r/(2a)\right)} \quad -(11)$$

$$\boxed{\psi(2p_z) = \frac{1}{2}\left(\frac{3}{\pi}\right)^{1/2} \cos\theta \cdot \left(\frac{1}{a}\right)^{3/2}\left(\frac{1}{2\sqrt{6}}\right)\frac{r}{a} \exp\left(-r/(2a)\right)} \quad -(12)$$

$$\boxed{\psi(2p_x) = -\frac{1}{2}\left(\frac{3}{2\pi}\right)^{1/2} \sin\theta\, e^{i\phi} \cdot \left(\frac{1}{a}\right)^{3/2}\left(\frac{1}{2\sqrt{6}}\right)\frac{r}{a} \exp\left(-r/(2a)\right)} \quad -(13)$$

$$\boxed{\psi(2p_y) = \frac{1}{2}\left(\frac{3}{2\pi}\right)^{1/2} \sin\theta\, e^{-i\phi} \cdot \left(\frac{1}{a}\right)^{3/2}\left(\frac{1}{2\sqrt{6}}\right)\frac{r}{a} \exp\left(-r/(2a)\right)} \quad -(14)$$