THE LAMB SHIFT IN ATOMIC HYDROGEN CALCULATED FROM EINSTEIN CARTAN EVANS (ECE) FIELD THEORY.

by

Myron W. Evans

Civil List Scientist

(www.aias.us and emyrone@aol.com)

and

Horst Eckardt,

Fellow of the Alpha Institute for Advanced Study

(horsteck@aol.com).

ABSTRACT

The Lamb shift in atomic H is calculated from the point of view of a generally covariant unified field theory - the Einstein Cartan Evans (ECE) field theory. The method adapted is to use an averaged vacuum potential that reproduces the g factor of the electron to experimental precision. This averaged potential is deduced from the Thomson radius of the photon and is used in the Schrodinger equation of atomic hydrogen (H) to increase the kinetic energy operator. The effect on the total energy levels of the H atom is then calculated and the Lamb shift deduced analytically without perturbation theory. It is found that the averaged vacuum energy affects each orbital of H in a different way through its effect on the electron proton distance. This effect is measured by distance $r(vac)$ that is found by comparison with the experimental Lamb shift between the 2s and 2p orbitals of H. This is a generally applicable method that removes the problems of quantum electrodynamics.

Keywords: Einstein Cartan Evans (ECE) unified field theory, Lamb shift in atomic H.

Paper 85 of ECE Series

1.

1. INTRODUCTION

The Einstein Cartan Evans (ECE) unified field theory {1-12} is generally covariant in all its sectors and is a causal and objective field theory of physics. As such it is diametrically the opposite of quantum electrodynamics (QED), the method usually used to calculate the Lamb shift in atomic hydrogen (H) {13 - 15}. The QED method relies on hidden assumptions and adjustable parameters, so it is not a first principles theory. QED is a theory of special relativity which at the same time acausal and subjective, and violates special relativity, being therefore internally inconsistent on a basic level. The reason is that QED assumes that a point electron radiates a virtual photon that does not obey the Einstein equation of special relativity and cannot be observed experimentally. The virtual photon produces a virtual electron and a virtual positron, which again violate special relativity. These virtual electrons and positrons can go backwards or forwards in time, travel at any speed, and so on {13 - 15}. The point electron is affected by these virtual entities and is said to be shielded. These ideas emerged from the fact that QED is based on the Dirac equation of an electron and the equation of the quantized electromagnetic field, in which the potential is subject to second quantization and expressed in terms of creation and annihilation operators. A degree of precision is claimed for QED, but this is unjustifiable, because it is based on methods used to remove singularities in a perturbation method. These methods include mass re-normalization, dimensional regularization, and re-normalization of singularities introduced by perturbation theory; the arbitrary cut off of a series expansion, and the use of thousands of terms, usually put into diagrammatic form and known as Feynman diagrams. The result of the QED Lamb shift calculation is expressed in terms of the fine structure constant (α). The latter's precision is however determined by its least precise constant, the Planck constant h. The experimentally measured relative standard uncertainty of h { 16 } is 1.7×10^{-7} and this is orders of magnitude greater than that of the electron g factor, which is 7.5×10^{-12}

2

So QED can never produce the g factor to a precision greater than that of the Planck constant.

In Section 2, the ECE theory is used to calculate the Lamb shift in atomic H in the non-relativistic limit described by the Schrödinger equation. The average effect of the vacuum is deduced from the Thomson radius of the photon. This method has previously been used in ECE theory {1-12} to produce the g factor of the electron to the limit of precision of the fine structure constant (the precision determined by h as just argued). The g factor of the electron is increased in ECE theory as follows:

$$ g \rightarrow g \left(1 + \frac{\alpha}{4\pi} \right)^2 . \quad\quad - (1) $$

To calculate the Lamb shift with the Schrödinger equation, the kinetic energy operator is increased by the same amount as in Eq. (1). This vacuum effected kinetic energy operator, when used in the Schrödinger equation of the H atom, produces tiny changes in the electron to proton distance in each orbital of the H atom. The change is different for each orbital and is expressed in terms of the distance r $\left(vac \right)$. The latter can be calculated analytically without perturbation theory. This method is first shown to produce the basic feature of the Lamb shift in atomic H, and is then refined analytically.

In Section 3 the results are graphed and used to illustrate the fact that this method is a generally applicable one, capable of being refined almost indefinitely by contemporary analytical and computational software.

2. A SIMPLE NON-RELATIVISTIC CALCULATION OF THE LAMB SHIFT

The interaction of the electromagnetic field with the hydrogen atom is in ECE theory a problem of causal general relativity. In the non-relativistic quantum approximation to ECE theory {1-12} the electron and proton are bound together by a Coulomb potential and described by the well known Schrödinger equation of atomic H { 17 }:

$$\left(-\frac{\hbar^2 \nabla^2}{2m} - \frac{e^2}{4\pi\epsilon_0 r}\right)\phi = E\phi. \quad -(2)$$

Here $\hbar$ is the reduced Planck constant, m is the reduced mass, -e is the charge on the electron, e is te charge on the proton, ϵ_0 is the vacuum permittivity and r is the proton to electron distance and radial coordinate of the spherical polar system of coordinates. The total energy levels E of the H atom are given by the Schrödinger equation as:

$$E = \frac{\alpha}{2n^2 a} \quad cm^{-1} \ (wavenumbers) \quad -(3)$$

where a is the Bohr radius:

$$a = \frac{4\pi\epsilon_0\hbar^2}{m e_e^2} \quad -(4)$$

and where α is the fine structure constant

$$\alpha = \frac{e^2}{4\pi\hbar c \epsilon_0}. \quad -(5)$$

Here n is the principle quantum number { 17 } and c is the speed of light in a vacuum. The Schrödinger equation produces the result:

$$E(2s) = E(2p) = \frac{\alpha}{8a} \quad cm^{-1} \quad -(6)$$

i.e. the energy levels of the 2s orbital and the three 2p orbitals are the same, i.e. degenerate. The Dirac equation of special relativistic quantum mechanics refines this result { 18 } to show that:

$$E(2s_{1/2}) = E(2p_{1/2}) \quad -(7)$$

The observed Lamb shift is { 17 }:

$$E\left(2S_{1/2}\right) - E\left(2P_{1/2}\right) = 0.0353 \text{ cm}^{-1} \quad -(8)$$

and from Eqs. (6) and (8) produces a relative change:

$$\left(\frac{\Delta\bar{\nu}}{\bar{\nu}}\right)_{exp.} = 2.048 \times 10^{-7} \quad -(9)$$

i.e. about two parts in ten million. The Lamb shift has no explanation in the Schrödinger and Dirac equations of the unperturbed H atom. The generally accepted explanation for the Lamb shift was first given by Bethe { 19 } as the effect of the quantized electromagnetic field's zeroth energy level. This is known somewhat misleadingly as the effect of the vacuum. More precisely it is due to the way in which the electromagnetic field is quantized in terms of harmonic oscillators. In order to approach this problem in ECE theory an electromagnetic potential is used to increase the value of the kinetic energy operator of Eq. (2). This method was first used {1- 12} to calculate the g factor of the electron, and is based as follows on the Thomson radius of the photon.

The basic hypothesis is that:

$$\frac{\langle e A(vac)\rangle}{\hbar \kappa} = \frac{\alpha}{4\pi} \quad -(10)$$

where $A(vac)$ is the classical "vacuum potential", a term that is intended to denote the average effect of the electric and magnetic fields that exist in the absence of any photons. This idea comes from the fact that the lowest (or zero'th) energy level of the electromagnetic field quantized as a harmonic oscillator is

$$E = \left(n + \frac{1}{2}\right)\hbar\omega \quad -(11)$$

where $\hbar\omega$ is the quantum of energy and n the number of photons. It is seen that there is

an energy:

$$E_0 = \frac{1}{2}\hbar\omega \quad -(12)$$

when there are no photons (n = 0). This means { 17 } that there are fluctuating electric and magnetic fields present when there are no photons present. The linear momentum associated with these "vacuum fields" is denoted $p(\text{vac})$, and the linear momentum associated with $\hbar\omega$ is $\hbar\kappa$, where:

$$\kappa = \frac{\omega}{c} \quad -(13)$$

is the wave-number magnitude. These ideas come from the fact that in quantum mechanics { 17 } the zero'th energy eigenvalue of the harmonic oscillator is non-zero.

The magnitude of $A(\text{vac})$ may be expressed in terms of the magnetic flux density magnitude $B^{(o)}$ associated with the quantum of energy $\hbar\omega$ { 1-12 }. Thus

$$B^{(o)} = \frac{\mu_0 c e}{Ar} \quad -(14)$$

where μ_0 is the vacuum permeability defined by:

$$\epsilon_0 \mu_0 = \frac{1}{c^2} \quad -(15)$$

and where Ar is the surface area associated with the energy quantum $\hbar\omega$. This is obtained from the Thomson radius:

$$r = \frac{1}{\kappa} \quad -(16)$$

and so the surface area of the sphere with this radius is:

$$Ar = 4\pi r^2 = \frac{4\pi}{\kappa^2}. \quad -(17)$$

6.

Thus:

$$B^{(o)} = \kappa \left| A(vac) \right| = \kappa A^{(o)} = \frac{\mu_o c e \kappa^2}{4\pi} \qquad -(18)$$

and:

$$e A^{(o)} = \frac{e^2 \kappa}{4\pi \epsilon_o c} \qquad -(19)$$

The fine structure constant is therefore defined by:

$$\alpha = \frac{e A^{(o)}}{\hbar \kappa} = \frac{e^2}{4\pi \epsilon_o \hbar c} \qquad -(20)$$

The average value of $A^{(o)}$ is now found by averaging over the polar angles: recall that the

surface area of a sphere of radius r is:

$$S = \int_o^{2\pi} d\phi \int_o^{\pi} r^2 \sin\theta \, d\theta = 4\pi r^2 \qquad -(21)$$

so for unit radius:

$$\int_o^{2\pi} d\phi \int_o^{\pi} \sin\theta \, d\theta = 4\pi \qquad -(22)$$

Thus:

$$\langle A^{(o)} \rangle = \int_o^{2\pi} \int_o^{\pi} A^{(o)}(\theta, \phi) \, d\phi \sin\theta \, d\theta = 4\pi A^{(o)} \qquad -(23)$$

This procedure means that the quantum of momentum $\hbar \kappa$ with Thomson radius $1/\kappa$

is averaged over θ and ϕ of the spherical polar coordinate system, i.e. $A^{(o)}$

is averaged over all possible orientations. The result is Eq. (10), which comes from a

quantized electromagnetic field.

Another method of justifying the factor 4π is to consider the magnetic flux

associated with the quantum of electromagnetic energy $\hbar \omega$ {1-12}:

$$\overline{\Phi}^{(0)} := \frac{\mu_0 c e}{4\pi} = \frac{e}{4\pi \epsilon_0 c} \qquad -(24)$$

it is known { 13 } that the quantum of magnetic flux can also be expressed as:

$$\overline{\Phi} = \frac{\hbar}{e} \qquad -(25)$$

so:

$$\overline{\Phi}^{(0)} \propto \overline{\Phi}. \qquad -(26)$$

In fact:

$$\frac{e}{4\pi \epsilon_0 c} = \alpha \frac{\hbar}{e} \qquad -(27)$$

so

$$\overline{\Phi}^{(0)} = \alpha \overline{\Phi} \qquad -(28)$$

Q. E. D.

From Eq. (10) we may express $\langle A^{(0)} \rangle$ as follows:

$$\langle A^{(0)} \rangle = \frac{e}{16\pi^2 \epsilon_0 \hbar c^2} \langle E^{(0)} \rangle \quad -(29)$$

where $E^{(0)}$ is the zero'th point energy associated with the quantized electromagnetic field.

Now extend Eq. (29) to energy momentum:

$$\langle e A_\mu (vac) \rangle = \frac{\alpha}{4\pi} \langle p_\mu (vac) \rangle. \quad -(30)$$

It is seen that this is a type of minimal prescription { 13 - 15 } that accounts for the vacuum

energy. More accurately, vacuum energy should be termed space-time energy. The effect of

$\langle e A_\mu (vac) \rangle$ is as follows:

$$p_\mu \rightarrow \left(1 + \frac{\alpha}{4\pi}\right) p_\mu. \quad - (31)$$

So the Dirac equation of the electron changes to {1-12}:

$$\left. \left(\gamma^\mu p_\mu (1 + \alpha') - mc \right) \psi = 0, \atop \alpha' := \frac{\alpha}{4\pi}. \right\} \quad - (32)$$

This is equivalent to an increase in the Dirac matrix:

$$\gamma^\mu \rightarrow (1 + \alpha') \gamma^\mu. \quad - (33)$$

In QED this increase is denoted:

$$\gamma^\mu \rightarrow \left(1 + \Lambda^{(2)}\right) \gamma^\mu \quad - (34)$$

where $\Lambda^{(2)}$ is a convergent vertex { 13 - 15 }, a kind of adjustable parameter.

The well known factor 2 of the Dirac theory of the electron interacting with a classical electromagnetic field originates geometrically in the relation between the Minkowski metric $\eta^{\mu\nu}$ and the Dirac matrices { 13 }:

$$2\eta^{\mu\nu} = \gamma^\mu \gamma^\nu + \gamma^\nu \gamma^\mu. \quad - (35)$$

Both Dirac matrices change according to Eq. (33), i.e.:

$$\gamma^\mu \rightarrow \left(1 + \alpha'\right) \gamma^\mu, \quad - (36)$$
$$\gamma^\nu \rightarrow \left(1 + \alpha'\right) \gamma^\nu, \quad - (37)$$

so:

$$\gamma^\mu \gamma^\nu + \gamma^\nu \gamma^\mu \rightarrow 2(1+\alpha')^2 (\gamma^\mu \gamma^\nu + \gamma^\nu \gamma^\mu) \quad - (38)$$

and the g factor of the electron, g = 2, changes to:

$$g(ECE) = 2\left(1 + \frac{\alpha}{4\pi}\right)^2 . \quad - (39)$$

Note that this result happens to agree exactly to first order in α with the well known

calculation of Schwinger { 13 } in QED:

$$g(Schwinger) = 2 + \frac{\alpha}{\pi} \quad - (40)$$

and gives a small correction to second order in α .

The Schrödinger equation (α) is changed therefore by the same factor,

appearing in the kinetic energy operator. The latter is defined by the classical:

$$T = \frac{p^2}{2m} \quad - (41)$$

and the operator equation:

$$p \rightarrow -i\hbar \underline{\nabla} . \quad - (42)$$

So the kinetic energy operator is:

$$\hat{T} = -\frac{\hbar^2 \nabla^2}{2m} . \quad - (43)$$

This appears as the square of momentum, so:

$$\nabla^2 \rightarrow \left(1 + \frac{\alpha}{4\pi}\right)^2 \nabla^2 \quad - (44)$$

Our method of calculating the Lamb shift is therefore based on the same method as used to

calculate g of the electron {1-12}. Before proceeding, we discuss the precision of the ECE method for g of the electron.

From ECE theory:

$$g = 2\left(1 + \frac{\alpha}{4\pi}\right)^2 = 2 + \frac{\alpha}{\pi} + \frac{\alpha^2}{8\pi^2} \quad - (45)$$

with basic definition:

$$\alpha := \frac{e^2}{4\pi\epsilon_0 \hbar c} \quad - (45a)$$

The fundamental constants of physics are agreed upon by treaty in standards laboratories such as www.nist.gov of the US National Institute for Standards and Technology. This site gives:

$$g(\text{expt.}) = 2.0023193043718 \pm 0.0000000000075 \quad - (46)$$
$$\hbar(\text{expt.}) = (6.6260693 \pm 0.0000011) \times 10^{-34} \; Js \quad - (47)$$
$$e(\text{expt}) = (1.60217653 \pm 0.00000014) \times 10^{-19} \; C \quad - (48)$$
$$c(\text{exact}) = 2.99792458 \times 10^{8} \; ms^{-1} \quad - (49)$$
$$\epsilon_0(\text{exact}) = 8.854187817 \times 10^{-12} \; J^{-1}C^2 m^{-1} \quad (50)$$
$$\mu_0(\text{exact}) = 4\pi \times 10^{-7} \; Js^2 C^{-2} m^{-1} \quad - (51)$$

with relative standard uncertainties. By treaty:

$$\mu_0 \epsilon_0 = \frac{1}{c^2} \quad - (52)$$

is taken as exact. With a sufficiently precise value of π :

$$\pi = 3.141592653590. \quad - (53)$$

Using these values in Eq. (45a) gives:

$$\alpha = 0.0072997(34) \quad - (54)$$

11

where the result has been rounded off to the relative standard uncertainty of h, the least precisely known constant of α. Thus α cannot be more precisely known than h. The latter is determined by an experimental method described on www.nist.gov. Now use Eq. (54) in Eq. (45) to give the theoretical value of g from ECE theory:

$$g(ECE) = 2.0023323(49). \quad - (46)$$

The experimental value of g is known to much greater precision than h, and is:

$$g(xptl.) = 2.0023193043718 \pm 0.000\,000\,000\,0075. \quad - (47)$$

It is seen that the difference is:

$$g(ECE) - g(xptl.) = 0.0000004 \quad - (48)$$

which is about the same order of magnitude as the experimental uncertainty in h. It is concluded that ECE gives g as precisely as the uncertainty of h will allow. The latter is not given consistently in the literature. For example, a much used text such as Atkins { 17 } gives it as:

$$h(Atkins) = 6.62618 \times 10^{-34} \; Js \quad - (49)$$

without uncertainty estimates. This is very different (fourth decimal place) from the NIST value. Despite this, Atkins { 17 } gives:

$$g(Atkins) = 2.0023193 14 \quad - (50)$$

which is different from Eq. (47) only in the eighth decimal place. This is severely self-inconsistent. Atkins gives the g factor of the electron as:

12

$$g(\text{Atkins}) = 2.0023193114 \quad -(51)$$

which is different from the NIST value in the eighth decimal place, while it is claimed at NIST that $g(\text{exp})$ from Eq. (47) is accurate to the twelfth decimal place. This is a another major inconsistency between well used current sources. Ryder { 13 } gives:

$$g(\text{Ryder}) = 2.0023193048 \quad -(52)$$

which is different form the NIST value in the tenth decimal place, and Ryder claims that QED gives g to this same precision. As we have seen, this cannot be true because the uncertainty of any theoretically calculated g is limited by the uncertainty on h. In summary:

$$g(\text{Schwinger}) = 2 + \frac{\alpha}{\pi} = 2.0023322(8) \quad -(53)$$

$$g(\text{ECE}) = 2 + \frac{\alpha}{\pi} + \frac{\alpha}{8\pi^2} = 2.0023323(49) \quad -(54)$$

$$g(\text{NIST}) = 2.00231930437118 \pm 0.00000000075 \quad -(55)$$

$$g(\text{Atkins}) = 2.0023193114 \pm (?) \quad -(56)$$

$$g(\text{Ryder}) = 2.0023193048 \pm (?) \quad -(57)$$

and there is little doubt that other textbooks and sources will give further different values of g. So where does this finding leave the claims of QED? The Wolfram site claims that QED gives g using the series:

$$g = 2\left(1 + \frac{\alpha}{2\pi} - 0.328\left(\frac{\alpha}{\pi}\right)^2 + 1.181\left(\frac{\alpha}{\pi}\right)^3 - 1.510\left(\frac{\alpha}{\pi}\right)^4 + \dots + 4.393 \times 10^{-12}\right) \quad -(58)$$

which is derived from thousands of Feynman diagrams (sic). However, the various terms in eq. (58) come from all the assumptions listed in the introduction, and so this cannot be a

first principles theory despite its elaborate nature.

An even worse internal self-inconsistency emerges because at NIST the fine structure constant is claimed to be:

$$\alpha\left(NIST\right) = \left(7.297352560 \pm 0.000000024\right) \times 10^{-3} \quad -(59)$$

both experimentally and theoretically. The first thing to note is that this claim, Eq (59), is different in the eighth decimal place from Eq. (54), which is calculated from NIST's own fundamental constants, Eqs. (46) to (51). So the NIST website is internally inconsistent to an alarming degree, because it is at the same time claimed that Eq. (59) is accurate to the tenth decimal place. From Eq. (47) it is seen that h at NIST is accurate only to the sixth decimal place, which limits the accuracy of α to this, i.e. ten thousand times less precise than claimed. The source of this very large internal discrepancy must therefore be found. The theoretical claim for the fine structure constant at NIST comes from QED, which is described as a theory in which an electron emits a virtual photon, which emits virtual electron positron pairs. The virtual positron is attracted and the virtual electron is repelled from the real electron. This process results in a screened charge, and the fine structure constant in QED is the square of an entity known as the completely screened charge, a mathematical limiting value. This is defined as the limit of zero momentum transfer or in the limit of infinite distance. At high energies the fine structure constant drops to 1 / 128, and so is not a constant at all. It certainly cannot be claimed to be precise to the relative standard uncertainty of Eq. (59), taken directly from the NIST website itself (www.nist.gov).

The experimental claim for the fine structure constant at NIST comes from the use of the quantum Hall effect combined with a calculable cross capacitor to measure the standard resistance. The immediate problem with this method is the use of the von Klitzing

14

constant:

$$R_K = \frac{\hbar}{e^2} = \frac{\mu_0 c}{2} \quad (sic) \quad - (60)$$

which is clearly limited once more to the accuracy of h. The latter is found at NIST by a different experimental method, giving Eq. (47). The accuracy of the charge on the proton is only ten times better than h, so R_K cannot be more accurate than h. If α were really as accurate as claimed in Eq. (59), then R and h/e^2 would also be known to this accuracy, conflicting with Eqs. (47) and (48) of the same NIST website.

In view of these serious problems it is considered that the claim of QED to reproduce the Lamb shift to high accuracy is also in complete doubt, as are all the so-called "precision tests" of QED, for example the g of the electron, the Lamb shift, positronium and so forth. The only sensible way forward is to calculate the Lamb shift in terms of r(vac) and to find r(vac) from the experimental value of the Lamb shift, making no unjustifiable claims of precision. This aim is achieved as follows on the non-relativistic level and can be considerably refined in future work.

The distance r(vac) is defined from Eq. (2) by: and (44)

$$- \frac{\hbar^2 \nabla^2}{2m} \left(\frac{\alpha}{2\pi} + \frac{\alpha^2}{16\pi^2} \right) \psi = \frac{e^2}{4\pi \epsilon_0 r(vac)} \psi \quad - (61)$$

i.e. the effect of the vacuum potential is considered to be a shift in the electron to proton distance for each orbital. To first order in α :

$$\nabla^2 \psi = - \left(\frac{4mc}{\hbar} \right) \frac{1}{r(vac)} \psi. \quad - (62)$$

Using the normalization given by Atkins { 17 } we consider five of the H orbitals:

$$\phi(1s) = \frac{1}{\pi^{1/2}} \left(\frac{1}{a}\right)^{3/2} exp\left(-\frac{r}{a}\right), - (63)$$

$$\phi(2s) = \frac{1}{4\sqrt{2}\pi^{1/2}} \left(2-\frac{r}{a}\right) exp\left(-\frac{r}{2a}\right), - (64)$$

$$\phi(2p_z) = \frac{1}{2}\left(\frac{3}{\pi}\right)^{1/2} cos\theta \left(\frac{1}{a}\right)^{3/2} \left(\frac{1}{2\sqrt{6}}\right) \frac{r}{a} exp\left(-\frac{r}{2a}\right), - (65)$$

$$\phi(2p_x) = -\frac{1}{2}\left(\frac{3}{2\pi}\right)^{1/2} sin\theta e^{i\phi} \left(\frac{1}{a}\right)^{3/2} \left(\frac{1}{2\sqrt{6}}\right) \frac{r}{a} exp\left(-\frac{r}{2a}\right),$$
$$- (66)$$

$$\phi(2p_y) = \frac{1}{2}\left(\frac{3}{2\pi}\right)^{1/2} sin\theta e^{-i\phi} \left(\frac{1}{a}\right)^{3/2} \left(\frac{1}{2\sqrt{6}}\right) \frac{r}{a} exp\left(-\frac{r}{2a}\right).$$
$$- (67)$$

In spherical polar coordinates the laplacian operator is { 17 }:

$$\nabla^2\phi = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\phi}{\partial r}\right) + \frac{1}{r^2 sin\theta}\frac{\partial}{\partial\theta}\left(sin\theta\frac{\partial\phi}{\partial\theta}\right) + \frac{1}{r^2 sin^2\theta}\frac{\partial^2\phi}{\partial\phi^2}.$$
$$- (68)$$

Computer algebra may now be used to evaluate r(vac) for each orbital. We first consider for

the sake of illustration a special case of the 2p$_z$ orbital, the maximum:

$$cos\theta = 1. \qquad - (68a)$$

It is found that:

$$\frac{1}{r(vac)}(1s) = \frac{1}{2\pi^{1/2}} \cdot \frac{\not{L}}{2nca}\left(\frac{1}{r} - \frac{1}{2a}\right), \quad -(69)$$

$$\frac{1}{r(vac)}(2s) = \frac{1}{2\pi^{1/2}} \cdot \frac{\not{L}}{2nca}\left(\frac{1}{r} - \frac{1}{8a}\right), \quad -(70)$$

$$\frac{1}{r(vac)}\left(2p_z, \cos\theta=1\right) = \frac{1}{2\pi^{1/2}} \cdot \frac{\not{L}}{2nca}\left(\frac{1}{r} - \frac{1}{8a} - \frac{a}{r^2}\right).$$
$$-(71)$$

This checks that the method gives the correct qualitative result. A remarkable simplification

occurs to show that the Lamb shift is proportional to:

$$\frac{1}{r(vac)}(2s) - \frac{1}{r(vac)}\left(2p_z, \cos\theta=1\right) = \frac{1}{4\pi^{1/2}} \cdot \frac{\not{L}}{nc} \cdot \frac{1}{r^2}.$$
$$-(72)$$

More generally the Lamb shift will depend on the polar coordinates. Despite its huge

complexity, QED is unable to give these detailed results, which are found by dispensing with

perturbation theory and virtual particles, and calculating analytically. Finally the constant of

proportionality in Eq. (72) has to be found. The potential energy of the H atom in wave-

numbers is:

$$V_0 = -\frac{\alpha}{r} \qquad -(73)$$

before the vacuum perturbation and is:

$$V = -\frac{\alpha}{r + r(vac)} \qquad -(74)$$

after the vacuum perturbation. So the change in potential energy is positive:

$$\Delta V = \left|V - V_0\right| = \alpha\left(\frac{1}{r} - \frac{1}{r + r(vac)}\right)$$
$$= \alpha\left(\frac{r(vac)}{r(r + r(vac))}\right) \qquad -(75)$$

17

The relative change in potential energy is:

$$\Delta V / V_o = \frac{r(vac)}{r + r(vac)} \qquad -(76)$$

which produces a proportional change in total energy of:

$$\Delta E = \frac{\alpha}{2n^2 a}\left(\frac{r(vac)}{r + r(vac)}\right) \cdot \qquad -(77)$$

The relative change in total energy is therefore:

$$\frac{\Delta E}{E} = \frac{r(vac)}{r + r(vac)} = \left(1 + \frac{r}{r(vac)}\right)^{-1} \qquad -(78)$$

between the 2s and 2p$_z$ orbital this produces the relative change:

$$\Delta\left(\frac{\Delta E}{E}\right) = 2.048 \times 10^{-7} = \left(1 + \frac{r}{r(vac)(2s)}\right)^{-1} - \left(1 + \frac{r}{r(vac)(2p_z}\right)^{-1} \qquad -(79)$$

from which r is calculated to be:

$$r = 1.12 \times 10^{-20} \, m.$$

This is a remarkable result because it shows why the Lamb shift is constant as observed. For a

given orientation defined by:

$$\cos\theta = 1 \qquad -(81)$$

Lamb
the/shift is determined completely by 1 / r within a constant of proportionality

$$\Delta\left(\frac{\Delta E}{E}\right) = 4\pi^{1/2} \frac{\hbar}{mc} r. \qquad -(82)$$

The next section discusses the angular dependence of the Lamb shift and gives some

graphical results.