85(5): Derivation of the Fine Structure Constant.

The basic hypothesis of paper 18 (Chapt. 20 of vol. 1) is:

$$\frac{\alpha}{4\pi} = \frac{eA^{(vac)}}{\hbar \kappa} \quad -(1)$$

where α is the fine structure constant, $A^{(vac)}$ ~~the~~ the vacuum potential and κ the wavenumber. In this note this hypothesis is justified using Eq. (3.127) of "The Enigmatic Photon", volume 4 - see Omnia Opera of www.aias.us. This is, for one photon:

$$B^{(0)} = \frac{\mu_0 c e}{Ar} \quad -(2)$$

where:

$$Ar = \frac{4\pi}{\kappa^2} \quad -(3)$$

is the surface area of a photon, modelled as a sphere of radius:

$$r = \frac{1}{\kappa} \quad -(4)$$

sometimes known as the Thomson radius. The surface area of a sphere of radius r is $4\pi r^2$.

So:

$$B^{(0)} = \kappa A^{(0)} = \frac{\mu_0 c e \kappa^2}{4\pi} \quad -(5)$$

and

$$\boxed{eA^{(0)} = \frac{\mu_0 c e^2 \kappa}{4\pi} = \frac{e^2 \kappa}{4\pi \epsilon_0 c}} \quad -(6)$$

2)

Therefore:

$$\boxed{\frac{eA^{(0)}}{\hbar\kappa} = \alpha = \frac{e^2}{4\pi\epsilon_0\hbar c}} \quad - (7)$$

Finally we:

$$A^{(vac)} = \frac{A^{(0)}}{4\pi} \quad - (8)$$

and:

$$\frac{e\,A^{(vac)}}{\hbar\kappa} = \frac{\alpha}{4\pi}. \quad - (9)$$

QED

Therefore it is possible to derive the fine structure constant if the magnetic flux of one photon is:

$$\Phi^{(0)} = A_r B^{(0)} = \mu_0 c e \quad - (10)$$

and if the surface area of the photon is defined by its Thomson radius ~~through~~ eq. (3). The meaning of the fine structure constant is, ~~that~~ ist is the ratio of momenta defined by eq. (7), provided that the photon volume is modelled by a sphere whose surface area is defined by its Thomson radius, $1/\kappa$.

3) In this case the ratio of $eA^{(0)}$ to $\hbar\kappa$ is α, a universal constant. Also, for such a photon, its magnetic flux $\Phi^{(0)}$ is also the universal constant defined in eq. (10). The fundamental flux is:

$$\Phi^{(0)} = \mu_0 c e = \frac{e}{\epsilon_0 c} \quad — (11)$$

using:

$$\mu_0 \epsilon_0 = \frac{1}{c^2}. \quad — (12)$$

A justification must now be found for the extra factor 4π in eq (8). This factor is justified to one part in a million precision by comparison with the electron g factor. Theoretical justification can be found by assuming that:

$$A^{(0)} = \langle A^{(0)} \rangle = 4\pi A^{(vac)}. \quad — (13)$$

The surface area of a sphere with radius r is:

$$S = \int_0^{2\pi} d\theta \int_0^{\pi} r^2 \sin\phi \, d\phi = 4\pi r^2 \quad — (14)$$

so:

$$\boxed{\int_0^{2\pi} d\theta \int_0^{\pi} \sin\phi \, d\phi = 4\pi} \quad — (15)$$

and

$$\langle A^{(0)} \rangle = \int_0^{2\pi} \int_0^{\pi} A^{(vac)}(\theta, \phi) \, d\theta \sin\phi \, d\phi \quad — (16)$$

4)

This means that a photon of Thomson radius $1/\kappa$ is averaged over θ and ϕ of the spherical polar coordinate system, i.e. $A^{(vac)}(\theta, \phi)$ is averaged over all possible orientations. Thus:

$$\frac{eA^{(0)}}{\hbar\kappa} = \alpha = \frac{e\langle A^{(0)}\rangle}{\hbar\kappa} = \frac{4\pi eA^{(vac)}}{\hbar\kappa} \quad -(1$$

and

$$\boxed{\frac{eA^{(vac)}}{\hbar\kappa} = \frac{1}{4\pi}\alpha} \quad -(18)$$

Q.E.D.

Another method of justifying the factor $1/4\pi$ is to use in eq. (11) the definition:

$$\Phi^{(0)} := \frac{\mu_0 c e}{4\pi} = \frac{e}{4\pi\epsilon_0 c} \quad -(19)$$

for a spherical photon of Thomson radius $1/\kappa$. It is known that the quantum of magnetic flux is

$$\Phi = \frac{\hbar}{e} \quad -(20)$$

So

$$\Phi^{(0)} \propto \Phi \quad -(21)$$

In fact:

$$\frac{e}{4\pi\epsilon_0 c} = \alpha\frac{\hbar}{e} \quad -(22)$$

So:

$$\boxed{\Phi^{(0)} = \alpha\,\Phi} \quad -(23) \quad \text{Q.E.D}$$