82(4): Interaction of Photon and Electron

The ECE equations for this interaction are:

$$(\square + kT)\, q^a_\mu = 0 \qquad -(1)$$

$$(\square + kT)\, A^a_\mu = 0 \qquad -(2)$$

Denote the electron momentum by p^μ and the photon momentum by π^μ. In eq. (1):

$$\square = -\frac{1}{\hbar^2} p^\mu p^\mu \qquad -(3)$$

In eq. (2):

$$\square = -\frac{1}{\hbar^2} \pi^\mu \pi_\mu \qquad -(4)$$

as a result of:

$$p^\mu = i\hbar\partial^\mu, \quad \pi^\mu = i\hbar\partial^\mu \qquad -(5)$$

These are equations of wave-particle dualism.

For the free electron:

$$\left(\square + \left(\frac{m_e c}{\hbar}\right)^2\right) q^a_\mu = 0, \qquad -(6)$$

$$\square = -\frac{1}{\hbar^2} p^\mu p_\mu . \qquad -(6a)$$

For the free photon:

$$\left(\square + \left(\frac{m_p c}{\hbar}\right)^2\right) A^a_\mu = 0 \qquad -(7)$$

$$\square = -\frac{1}{\hbar^2} \pi^\mu \pi_\mu \qquad -(7a)$$

The interaction is described by:

$$p^\mu_{free} + \pi^\mu_{free} = p^\mu_{free} + eA^\mu + \pi^\mu_{free} - eA^\mu$$

$$-(8)$$

2)

i.e.:

$$p^\mu \to p^\mu + eA^\mu \quad - (9)$$

$$\pi^\mu \to \pi^\mu - eA^\mu \quad - (10)$$

In general A^μ is complex valued, so:

$$\Box_e \to \frac{1}{\hbar^2}\left(p^\mu + eA^\mu\right)\left(p_\mu + eA_\mu^*\right) - (11)$$

$$\Box_p \to \frac{1}{\hbar^2}\left(\pi^\mu - eA^\mu\right)\left(\pi_\mu - eA_\mu^*\right) - (12)$$

Substituting in eqs (1) and (2) and averaging out terms to first order:

$$\left(\Box_e - \frac{e^2 A^{(0)2}}{\hbar^2} + \left(\frac{m_e c}{\hbar}\right)^2\right) V_\mu^a = 0 - (13)$$

$$\left(\Box_p - \frac{e^2 A^{(0)2}}{\hbar^2} + \left(\frac{m_p c}{\hbar}\right)^2\right) A_\mu^a = 0 - (14)$$

So:

$$(kT)_e = \frac{1}{\hbar^2}\left(m_e^2 c^2 - e^2 A^{(0)2}\right) - (15)$$

$$(kT)_p = \frac{1}{\hbar^2}\left(m_p^2 c^2 - e^2 A^{(0)2}\right) - (16)$$

<u>For the electromagnetic field:</u>

$$\left(\Box_p + \left(\frac{m_p c}{\hbar}\right)^2\right) A_\mu^a = \left(\frac{e^2 A^{(0)2}}{\hbar^2}\right) A_\mu^a$$

3) For the fermion (electron) field:

$$\left(\Box_e + \left(\frac{m_e c}{\hbar} \right)^2 \right) q_\mu^a = \left(\frac{e^2 A^{(0)2}}{\hbar^2} \right) q_\mu^a \quad (18)$$

Eqs. (17) and (18) are those of generally covariant quantum electrodynamics.

Here:

$$A^{(0)2} = -i \left| \underline{A}^{(1)} \times \underline{A}^{(2)} \right| . \quad (19)$$

The classical equivalents are:

$$p^\mu p_\mu = m_e^2 c^2 - e^2 A^{(0)2} \quad (20)$$

$$\pi^\mu \pi_\mu = m_p^2 c^2 - e^2 A^{(0)2} \quad (21)$$

i.e.

$$p^\mu p_\mu - m_e^2 c^2 = \pi^\mu \pi_\mu - m_p^2 c^2 = - e^2 A^{(0)2} \quad (22)$$

The gravitational interaction requires the consideration of further terms in kT. The above is summarized in: