85(3): Comparison of Angular Momentum from the Hamilton Jacobi Method and Direct Integration Method.

The direct integration method of paper 81 gives:

$$J = \frac{1}{\gamma m \omega}\left(\gamma m \omega r_0 + eA^{(0)}\right)\left(\gamma m v_0 + eA^{(0)}\right) \quad -(1)$$

where r_0 and v_0 are the initial position and velocity of the electron. The Hamilton Jacobi method gives:

$$J = \frac{c e^2 A^{(0)2}}{\omega\left(m^2c^2 + e^2 A^{(0)2}\right)^{1/2}}. \quad -(2)$$

The interaction terms of eq. (1) are:

$$J_{int} = eA^{(0)}\left(r_0 + \frac{v_0}{\omega}\right) + \frac{e^2A^{(0)2}}{\gamma m \omega}. \quad -(3)$$

So we may compare eqns. (2) and (3).

Non-Relativistic Limit

This is:

$$\left.\begin{array}{l}\gamma \to 1, \\ mc \gg eA^{(0)},\end{array}\right\} \quad -(4)$$

and both eqns. (2) and (3) give:

$$\boxed{J \to \frac{e^2A^{(0)2}}{m\omega}} \quad -(5)$$

So both methods give the same result, Q.E.D.

Hyper-Relativistic Limit

This is:
$$eA^{(0)} \gg mc \quad -(6)$$
so eq. (2) becomes:
$$J = \frac{ecA^{(0)}}{\omega} \quad -(7)$$
If the second order term in eq. (3) is negligible, then
$$J_{int} = eA^{(0)}\left(r_0 + \frac{v_0}{\omega}\right) \quad -(8)$$
so:
$$r_0 + \frac{v_0}{\omega} \rightarrow \frac{c}{\omega} \quad -(9)$$
i.e.
$$\left.\begin{array}{l} v_0 \rightarrow c, \\ \frac{v_0}{\omega} \gg r_0. \end{array}\right] \quad -(10)$$

General Comparison

Comparing eqns. (2) and (3):
$$eA^{(0)}\left(r_0 + \frac{v_0}{\omega}\right) + \frac{e^2A^{(0)2}}{\gamma m\omega} = \frac{ce^2A^{(0)2}}{\omega\left(m^2c^2 + e^2A^{(0)2}\right)^{1/2}} \quad -(11)$$
If for an initially stationary electron at the origin:
$$v_0 = 0, \quad r_0 = 0 \quad -(12)$$
we obtain:
$$\gamma = \frac{1}{mc}\left(m^2c^2 + e^2A^{(0)2}\right)^{1/2} \quad -(13)$$

and this is precisely what was used in the Hamilton Jacobi method of chapter 12, volume one, of "The Enigmatic Photon" (Kluwer 1994 and 2002).

Q.E.D.

Interpretation of γ

In eq. (13), γ comes from the rest energy mc^2 of the electron and the additional electron momentum $eA^{(0)}$ imparted by the electromagnetic field. Thus:

$$\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-1/2} \quad -(14)$$

where v must be interpreted as the velocity of the electron, as distinct from u of the Lorentz transformation, where u is the speed of one frame with respect to another. So it is found that:

$$1 - \frac{v^2}{c^2} = \frac{1}{1 + \left(\frac{eA^{(0)}}{mc}\right)^2} \quad -(15)$$

For low velocity v compared with c:

$$\boxed{v \sim \frac{e^2 A^{(0)2}}{mc^2}} \quad -(16)$$

i.e. if $eA^{(0)} << mc$.

4)

Remarks

Eq. (16) means that the initially stationary electron has attained an orbital velocity of $e^2 A^{(0)2} / mc^2$ from the applied e/m field. So everything is self-consistent. The important result for the IFE and RFR effects is eqn. (5). Thus IFE and RFR for one electron have been derived in many different ways. They assume a circularly polarized e/m field. In the non-relativistic limit of the Prague experiments, the interaction kinetic energy is:

$$T = \frac{e^2 A^{(0)2}}{2m} \quad -(17)$$

and comparing eqns (5) and (17):

$$T = \frac{1}{2} \omega J = \frac{1}{2} \Omega J, \quad -(18)$$

i.e in this limit:

$$\boxed{\Omega = \omega} \quad -(19)$$

as also derived from the HJ method, Q.E.D.

RFR is then derived from eq. (17) using the SU(2) basis, so:

$$\boxed{\hbar \omega_{res} = e^2 A^{(0)2} / m} \quad -(20)$$

i.e. the difference of energy levels of the Pauli matrix