Notes 77(5): Evolution of the Spiral Arm

In the central bulge:

$$\Omega = \frac{d\theta}{dt} = \frac{v}{r} = \left(\omega^2 + \left(\frac{eB^{(0)}}{m}\right)^2\right)^{1/2} \quad -(1)$$

Therefore:

$$\theta = \int \frac{v}{r}\, dt = \int \left(\omega^2 + \left(\frac{eB^{(0)}}{m}\right)^2\right)^{1/2} dt \quad -(2)$$

Now define:

$$\omega = \frac{V_0}{r} \quad -(3)$$

So:

$$\boxed{\theta = \int \left(\left(\frac{V_0}{r}\right)^2 + \left(\frac{eB^{(0)}}{m}\right)^2\right)^{1/2} dt} \quad -(4)$$

This is the general relation between θ and r, with adjustable parameters V_0 and $B^{(0)}$.

In the limit:

$$\omega \gg eB^{(0)}/m \quad -(5)$$

$$\theta \rightarrow \int \frac{V_0 t}{r} := \frac{V_0 \tau}{r} \quad -(6)$$

giving the hyperbolic spiral.