77(2): Calculation of the Angle θ.

From eqs. (3) and (4) of note 77(1):

$$\tan\theta = -\frac{v_y}{v_x} \quad -(1)$$

$$\boxed{\theta = \tan^{-1}\left(-\frac{v_y}{v_x}\right) = \omega t} \quad -(2)$$

In a spiral galaxy it is observed that:

$$v = \left(v_x^2 + v_y^2\right)^{1/2} = \text{constant.} \quad -(3)$$

From eq (5) of note 77(1):

$$v = \frac{ecB^{(0)}}{m\omega} \quad -(4)$$

so ω is a constant of the motion. Therefore:

$$\boxed{t = \frac{1}{\omega}\tan^{-1}\left(-\frac{v_y}{v_x}\right)} \quad -(5)$$

If we denote:

$$\phi := -\frac{v_y}{v_x} \quad -(6)$$

then:

$$\boxed{t = \frac{1}{\omega}\left(\phi - \frac{\phi^3}{3} + \frac{\phi^5}{5} - \frac{\phi^7}{7} + \dots\right)} \quad -(7)$$

for appropriate limits on ϕ:

$$|\phi| < 1 \quad -(8)$$

in radians

2)

In order to have positive values of time t in eq. (7), the modulus of ϕ is needed:

$$t := \frac{1}{\omega}\left(|\phi| - \frac{|\phi|^3}{3} + \frac{|\phi|^5}{5} - \dots\right) \quad -(9)$$

For:

$$|\phi| << 1 \quad -(10)$$

$$\boxed{t \sim \frac{|\phi|}{\omega}} \quad -(11)$$

Interpretation

The time t represents a relativistic phenomenon, because it is derived from the relativistic HJ equation (i.e. the Einstein equation of special relativity with minimal prescription). In eq. (11):

$$t \sim \frac{1}{\omega}\left|\frac{v_Y}{v_X}\right| \quad -(12)$$

where

$$v_X^2 + v_Y^2 = v^2 \quad -(13)$$

If it is assumed that the observed orbital velocity is:

$$v = \text{constant} \quad -(14)$$

then t is determined by ω and v. Finally

$$\boxed{\theta = \omega t \sim \left|\frac{v_Y}{v_X}\right|} \quad -(15)$$

in this approximation