76(2): Rotational Kinetic Energy from the Intrinsic Cartan Torsion.

In the first approximation a galaxy cluster is considered to be made up of galaxies which are not moving with respect to each other. The cluster is then made up of n particles of mass m_α. Due to its intrinsic Cartan torsion the galaxy cluster rotates at angular velocity $\underline{\omega}$ about some point fixed in the cluster frame of reference. The cluster moves at velocity $\underline{V}$ with respect to ~~the~~ a fixed coordinate system. The instantaneous velocity of the α'th particle (galaxy) in the fixed system is (paper 55):

$$\underline{v}_f = \underline{V} + \underline{v}_r + \underline{\omega} \times \underline{r} . \quad -(1)$$

In the rigid galaxy cluster:

$$\underline{v}_r = \left(\frac{d\underline{r}}{dt} \right)_{\text{cluster}} = \underline{0} \quad -(2)$$

so in the cluster frame of reference:

$$\boxed{\underline{v}_\alpha = \underline{V} + \underline{\omega} \times \underline{r}_\alpha} . \quad -(3)$$

The kinetic energy of the α'th galaxy is:

$$T_\alpha = \frac{1}{2} m_\alpha v_\alpha^2 \quad -(4)$$

$$= \frac{1}{2} m_\alpha \left(\underline{V} + \underline{\omega} \times \underline{r}_\alpha \right)^2 - (5)$$

where m_α is the mass of the galaxy. The angular velocity $\underline{\omega}$ is due to the intrinsic Cartan torsion of the galaxy cluster.

2) The complete kinetic energy of the galaxy cluster is:

$$T_\alpha = \frac{1}{2}\sum_\alpha m_\alpha V^2 + \sum_\alpha m_\alpha \underline{V}\cdot\underline{\omega}\times\underline{r}_\alpha + \frac{1}{2}\sum_\alpha m_\alpha\left(\underline{\omega}\times\underline{r}_\alpha\right)^2 \quad -(6)$$

If the choice of origin vector $\underline{r}_\alpha$ is the same as the center of mass then:

$$\sum_\alpha m_\alpha \underline{V}\cdot\underline{\omega}\times\underline{r}_\alpha = \underline{V}\cdot\underline{\omega}\times\sum_\alpha m_\alpha r_\alpha = \underline{0}. \quad -(7)$$

The total kinetic energy of the galaxy cluster is then:

$$T = T_{trans} + \boxed{T_{rot}} \quad -(8)$$

where:

$$T_{trans} = \frac{1}{2}\sum_\alpha m_\alpha V^2 = \frac{1}{2} m V^2 \quad -(9)$$

$$T_{rot} = \frac{1}{2}\sum_\alpha m_\alpha\left(\underline{\omega}\times\underline{r}_\alpha\right)^2 \quad -(10)$$

Using vector analysis: $-(11)$

$$T_{rot} = \frac{1}{2}\sum_{i,j}\omega_i\omega_j\sum_\alpha m_\alpha\left(\delta_{ij}\sum_k\left(x_{\alpha,k}^2 - x_{\alpha,i}x_{\alpha,j}\right)\right)$$

3)

$$:= \frac{1}{2} \sum_{i,j} I_{ij}\, \omega_i\, \omega_j \, .$$

The moment of inertia of the galaxy cluster is:

$$\boxed{I_{ij} = \sum_{\alpha} m_\alpha \left(\delta_{ij} \sum_k \left(x_{\alpha,k}^2 - x_{\alpha,i}\, x_{\alpha,j} \right) \right)} \quad — (12)$$

where quantities are calculated in the cluster frame of reference (as in a molecule frame of reference for example for a symmetric, spherical, or asymmetric top).

So "missing mass" or "dark matter" is:

$$\boxed{T_{rot} = \frac{1}{2} \sum_{i,j} I_{ij}\, \omega_i\, \omega_j} \quad — (13)$$

The value of T_{rot} depends on the value of I_{ij} and of ω_i and ω_j. The origin of the angular velocities ω_i and ω_j is intrinsic Cartan torsion, unconsidered in the standard model.

We now follow paper 55.

4) In eq. (77) of paper 55 the generally covariant torque equation was defined in differential form notation:

$$N^a = c\,(d \wedge J^a + \omega^a{}_b \wedge J^b) \quad -(14)$$

where the angular momentum form is:

$$J^a_\mu = J^{(0)} q^a_\mu . \quad -(15)$$

In the classical, non-relativistic, limit the Euler equation of motion is (vector notation):

$$\underline{N} = \left(\frac{d\underline{J}}{dt}\right)_{fixed} = \left(\frac{d\underline{J}}{dt}\right)_{cluster} + \underline{\omega} \times \underline{J} . \quad -(16)$$

For any vector $\underline{Q}$:

$$\left(\frac{d\underline{Q}}{dt}\right)_{fixed} = \left(\frac{d\underline{Q}}{dt}\right)_{cluster} + \underline{\omega} \times \underline{Q} \quad -(17)$$

and the generally covariant $\underline{Q}$ is:

$$Q^a_\mu = Q^{(0)} q^a_\mu . \quad -(18)$$

In these definitions, the spin connection is:

$$\omega^a_{\mu b} = (\omega^a_{0b}, -\underline{\omega}^a{}_b) \quad -(19)$$

5) and for each a and b the angular velocity is:

$$\underline{\Omega}^a{}_b := c\,\underline{\omega}^a{}_b \quad -(20)$$

The units of the spin connection are m^{-1} and the units of the angular velocity $\underline{\Omega}^a{}_b$ are rad s^{-1}.
The spin part of the Cartan torsion of the galaxy cluster is given by eq. (90) of paper 55:

$$\underline{T}^a_s = \underline{\nabla} \times \underline{q}^a - \underline{\omega}^a{}_b \times \underline{q}^b \quad -(21)$$

and the spin-torque of the galaxy cluster is:

$$\underline{N}^a_s = c\,J^{(0)}\,\underline{T}^a. \quad -(22)$$

The <u>intrinsic angular velocity</u> of the galaxy cluster is:

$$\underline{\Omega}^a{}_b = c\,\underline{\omega}^a{}_b. \quad -(23)$$

If we consider the rotation as being independent of the translation:

$$\omega^a_{\mu b} = -\frac{\kappa}{2}\,\epsilon^a{}_{bc}\,V^c \quad -(24)$$

where κ is a characteristic wave-number of the galaxy cluster's rotation.

6) Adopting the complex circular basis eq. (21) becomes:

$$\underline{T}_s^{(1)*} = \underline{\nabla} \times \underline{q}^{(1)*} + i\kappa \underline{q}^{(2)} \times \underline{q}^{(3)}$$

et cyclicum — (25)

so: $$\underline{N}_s^{(1)*} = c J^{(0)} \left(\underline{\nabla} \times \underline{q}^{(1)*} + i\kappa \underline{q}^{(2)} \times \underline{q}^{(3)} \right)$$ — (26)

The wavenumber κ may be expressed as:

$$\kappa = \frac{\Omega}{v} \quad — (27)$$

where Ω is the angular velocity due to the spin torsion $\underline{T}_s^{(1)*}$.

These equations are ~~generally~~ covariant, so hold in any frame of reference. If applied in the galaxy cluster frame for each galaxy we obtain the rotational kinetic energy (13). This is due to the spin torsion $\underline{T}_s$, which is missing completely from the EH ~~theory~~. This Ω produces the orbital velocity:

$$\boxed{v = \Omega r} \quad — (28)$$

of an edge galaxy, where r is its distance from the centre of mass of the cluster.