76(1): Basis of the Dark Matter Hypothesis, the Virial Theorem

The whole of the hugely elaborate dark matter hypothesis is based on the virial theorem of classical dynamics. It is used to assume that the total kinetic energy should be half the total gravitational binding energy. For galaxy clusters, stars at the edge are found to have a much greater velocity than can be found from the virial theorem. It is then assumed that there is missing mass, and from that, a hugely elaborate theory of dark matter has emerged uncritically. In all of this, the Cartan torsion is never considered, only the weak field limit of the Cartan curvature. Therefore rotational kinetic energy is never considered. This is clear from the virial theorem, which is used conventionally as follows.

The Virial Theorem

The virial G of N point particles is defined as:

$$G = \sum_{k=1}^{N} \underline{p}_k \cdot \underline{r}_k \quad - (1)$$

where $\underline{r}_k$ and $\underline{p}_k$ are position and momentum of the k th particle. It may be proven that if the virial is approximately constant:

$$\langle T \rangle = -\frac{1}{2} \sum_{k=1}^{N} \langle \underline{F}_k \cdot \underline{r}_k \rangle \quad - (2)$$

where $\langle T \rangle$ is average kinetic energy and $\underline{F}_k$ is force.

2) If the inter-particle forces result from a potential energy $V(r)$ that is some power n of the inter-particle distance r, the virial adopts the simple form:

$$\langle T \rangle = \frac{n}{2} \langle V \rangle \quad -(3)$$

The kinetic energy can then be calculated for very complicated systems. The virial theorem is used to derive the equipartition theorem or to compute the Chandrasekhar limit for the stability of white dwarf stars.

<u>Proof of Eq. (3)</u>

The time derivative of the virial is:

$$\frac{dG}{dt} = \sum_{k=1}^{N} \frac{d\underline{p}_k}{dt} \cdot \underline{r}_k + \sum_{k=1}^{N} \underline{p}_k \cdot \frac{d\underline{r}_k}{dt} \quad -(4)$$

$$= \sum_{k=1}^{N} \underline{F}_k \cdot \underline{r}_k + 2 \langle T \rangle \quad -(5)$$

The average is defined by:

$$\left\langle \frac{dG}{dt} \right\rangle = \frac{1}{\tau} \int_0^\tau \frac{dG}{dt}\, dt = \frac{1}{\tau} \int_0^\tau dG = \frac{1}{\tau} \left(G(\tau) - G(0) \right) \quad -(6)$$

$$= \sum_{k=1}^{N} \langle \underline{F}_k \cdot \underline{r}_k \rangle_\tau + 2 \langle T \rangle_\tau . \quad -(7)$$

3)

If:
$$\left\langle \frac{dG}{dt} \right\rangle_\tau = 0, \quad - (8)$$

then:
$$\langle T \rangle_\tau = -\frac{1}{2} \sum_{k=1}^{N} \langle \underline{F}_k \cdot \underline{r}_k \rangle_\tau . \quad - (9)$$

Potential Energy in the Virial Theorem

The force $\underline{F}_k$ on particle k is the sum of all the forces from the other particles j:

$$\underline{F}_k = \sum_{j=1}^{N} \underline{F}_{jk} \quad - (10)$$

where $\underline{F}_{jk}$ is the force applied by particle j on particle k.

Thus:
$$\sum_{k=1}^{N} \underline{F}_k \cdot \underline{r}_k = \sum_{k=1}^{N} \sum_{j=1}^{N} \underline{F}_{jk} \cdot \underline{r}_k . \quad - (11)$$

No particle acts on itself, so:
$$\underline{F}_{jk} = \underline{0} \quad \text{if } j = k . \quad - (12)$$

So:
$$\sum_{k=1}^{N} \underline{F}_k \cdot \underline{r}_k = \sum_{k=1}^{N} \sum_{j<k} \underline{F}_{jk} \cdot \underline{r}_k + \sum_{k=1}^{N} \sum_{j>k} \underline{F}_{jk} \cdot \underline{r}_k \quad - (13)$$

$$= \sum_{k=1}^{N} \sum_{j<k} \underline{F}_{jk} \cdot (\underline{r}_k - \underline{r}_j) \quad - (14)$$

4)

From Newton's third law:

$$\underline{F}_{jk} = -\underline{F}_{kj}. \quad -(15)$$

Force is the gradient of potential energy, so:

$$\underline{F}_{jk} = -\underline{\nabla}_{rk} V = -\frac{dV}{dr}\frac{(\underline{r}_k - \underline{r}_j)}{r_{jk}}. \quad -(16)$$

This is equal and opposite to:

$$\underline{F}_{kj} = -\underline{\nabla}_{rj} V \quad -(17)$$

so:

$$\sum_{k=1}^{N} \underline{F}_k \cdot \underline{r}_k = \sum_{k=1}^{N} \sum_{j<k} \underline{F}_{jk} \cdot (\underline{r}_k - \underline{r}_j)$$

$$= -\sum_{k=1}^{N} \sum_{j<k} \frac{dV}{dr} \frac{(\underline{r}_k - \underline{r}_j)^2}{r_{jk}} = -\sum_{k=1}^{N} \sum_{j<k} \frac{dV}{dr} r_{jk}$$

$$-(18)$$

If it is assumed that:

$$V(r_{jk}) = \alpha r_{jk}^{n} \quad -(19)$$

then:

$$-\sum_{k=1}^{N} \underline{F}_k \cdot \underline{r}_k = \sum_{k=1}^{N} \sum_{j<k} \frac{dV}{dr} r_{jk} = \sum_{k=1}^{N} \sum_{j<k} n V(r_{jk})$$

$$:= n U \quad -(20)$$

5) If: $\left\langle \frac{dG}{dt} \right\rangle_\tau = 0$ — (21)

then:

$$\langle T \rangle_\tau = -\frac{1}{2} \sum_{k=1}^{N} \langle \underline{F}_k \cdot \underline{r}_k \rangle_\tau = \frac{n}{2} \langle U \rangle_\tau$$ — (22)

Q.E.D.

For gravitational and electrostatic attraction:

$$n = -1$$ — (23)

so:

$$\boxed{\langle T \rangle_\tau = -\frac{1}{2} \langle U \rangle_\tau}$$ — (24)

The gravitational binding energy is the negative of the gravitational potential energy. For a sphere of uniform density, the gravitational potential energy is:

$$U = \frac{3}{5} \frac{GM^2}{r}$$ — (25)

In the original missing mass hypothesis by Zwicky, the virial theorem (24) was applied to the Coma cluster of galaxies. He estimated the cluster's total mass based on the motion of galaxies near its edge.

b) He compared this mass estimate to one based on the number of galaxies and total brightness of the cluster. This gave 400 times more mass than expected from the total number of galaxies alone. He then concluded that the total gravitational mass of the visible galaxies in the cluster was far too small for such fast orbits at the edge of the cluster.

This was then called the missing mass.

In reaching this conclusion, eq. (24) was used, in which V is the total gravitational binding energy of the galaxies. This was calculated from eq. (20) using the Newton inverse square law, i.e. using the weak field limit of the Cartan curvature and second Bianchi identity.

So what is really happening is that the kinetic energy from eq. (24) is much smaller than the kinetic energy calculated from the orbital velocity of galaxies at the edge of the cluster. Rotational kinetic energy is never considered.

Therefore, the theory of galactic rotation curves must be reworked using the Cartan torsion. Galactic rotation curves are plots of velocity of rotation versus the distance from the galactic centre. The Cartan torsion must also be used to explain why "dark matter" is spherically symmetric about the centre of the galaxy. This is easily explained from the fact that the rotational kinetic energy:

$$T_{rot} = \frac{1}{2} I \omega^2 \quad - (26)$$

generated from the Cartan torsion is spherically symmetric.

So we need to calculate T_{rot} from the Cartan torsion as in paper 55. In eq. (26), I is the moment of inertia of the galaxy and ω is its angular velocity in radians per second. Here ω is generated from the inherent Cartan torsion of the galaxy clusters.