74(2): The Balance Condition in Magnetic Motors and Beltrami Solutions

It was found in note 74(1) that there exists a balance condition when the spacetime torque is not operable, and the magnetic assembly is static:

$$\langle \underline{\nabla} \times \underline{q}^{a} \rangle = \langle \underline{\omega}^{a}{}_{b} \times \underline{q}^{b} \rangle \quad -(1)$$

If it is assumed that $\underline{\omega}^{a}{}_{b}$ is dual to $\underline{q}^{c}$, and if the complex circular basis is introduced, eq (1) becomes:

$$\langle \underline{\nabla} \times \underline{q}^{(1)*} \rangle = -i\kappa \langle \underline{q}^{(2)} \times \underline{q}^{(3)} \rangle$$

$$\langle \underline{\nabla} \times \underline{q}^{(2)*} \rangle = -i\kappa \langle \underline{q}^{(3)} \times \underline{q}^{(1)} \rangle$$

$$\langle \underline{\nabla} \times \underline{q}^{(3)*} \rangle = -i\kappa_1 \langle \underline{q}^{(1)} \times \underline{q}^{(2)} \rangle$$

$$-(3)$$

where

$$\kappa \neq \kappa_1 \quad -(4)$$

in general.

Plane Wave Solutions

In this case:

2)

$$\underline{q}^{(1)} = \underline{q}^{(2)*} = \frac{1}{\sqrt{2}}\left(\underline{i} - i\underline{j}\right) \exp\left(i(\omega t - \kappa Z)\right) \quad -(5)$$

$$\underline{A}^{(1)} = \underline{A}^{(2)*} = A^{(0)} \underline{q}^{(1)} \quad -(6)$$

and:

$$\underline{q}^{(3)} = \underline{q}^{(3)*} = \underline{k} . \quad -(7)$$

We have:

$$\boxed{\left\langle \exp\left(i(\omega t - \kappa Z)\right) \right\rangle = 0.} \quad -(8)$$

and

$$-i\underline{q}^{(1)} \times \underline{q}^{(2)} = \underline{k} . \quad -(9)$$

So:

$$\left\langle \underline{\nabla} \times \underline{q}^{(3)} \right\rangle = \kappa_1 \left\langle \underline{q}^{(3)} \right\rangle$$

$$= \kappa_1 \underline{k}$$

$$= \kappa_1 \underline{q}^{(3)} \quad -(10)$$

The balance condition is therefore:

$$\boxed{\left\langle \underline{\nabla} \times \underline{q}^{(3)} \right\rangle = \kappa_1 \underline{q}^{(3)}} \quad -(11)$$

i.e.

$$\kappa_1 = 0 \quad -(12)$$

In this example therefore:

$$\langle \underline{\nabla} \times \underline{q}^a \rangle = \langle \underline{\omega}^a{}_b \times \underline{q}^b \rangle = \underline{0} \quad -(13)$$

The magnetic assembly remains static and there is no net torque.

Beltrami Solution

As discussed in ACP vol 119(2), pp. 250ff (available on www.aias.us) a balance condition of the type:

$$\underline{\nabla} \times \underline{q} = \kappa \, \underline{q} \quad -(14)$$

is for a Beltrami vector field $\underline{q}$. It is one of the three basic types of field, solenoidal, complex lamellar and Beltrami. The quantity $\underline{\nabla} \times \underline{q}$ is analogous with vorticity, and $\underline{q}$ is analogous with velocity field, flux, or a Magnus streamline. The Beltrami field is a force-free fluid flow:

$$\underline{q} \times (\underline{\nabla} \times \underline{q}) = \underline{0} \quad -(15)$$

4)

In a Beltrami field:

$$\kappa = \frac{1}{q^2} \underline{q} \cdot \underline{\nabla} \times \underline{q} \quad -(16)$$

Beltrami fields are used in solar flare models, spiral galaxies, plasma vortex filaments and superconductivity.

Taking the curl of both sides of eq. (14) and assuming that:

$$\underline{\nabla} \cdot \underline{q} = 0 \quad -(17)$$

the Helmholtz equation is obtained:

$$\boxed{\nabla^2 \underline{q} + \kappa^2 \underline{q} = \underline{0}} \quad -(18)$$

The balance condition in this case is the Helmholtz equation.

This means that when the magnet is not rotating, spacetime is described by a Helmholtz equation for $\underline{q}$.

5) As described in ACP, vol 119(2), pp. 255 ff the wavenumber κ becomes complex valued in conducting media, so:

$$\underline{\nabla} \times \underline{q} = i\kappa'' \underline{q} \qquad (19)$$

where:

$$\kappa = \kappa' + i\kappa'' \qquad (20)$$

Taking the curl of eq. (19):

$$\underline{\nabla} \times (\underline{\nabla} \times \underline{q})$$
$$= \underline{\nabla} \times (\underline{\nabla} \cdot \underline{q}) - \nabla^2 \underline{q} = -\kappa''^2 \underline{q} \qquad (21)$$

i.e.

$$\nabla^2 \underline{q} = \kappa''^2 \underline{q} \qquad (22)$$

if

$$\underline{\nabla} \cdot \underline{q} = 0 \qquad (23)$$

The covariant form of eq. (22) is:

$$\Box q^a_\mu = -\kappa''^2 q^a_\mu \qquad (24)$$

which is the ECE wave equation if:

$$\boxed{kT = \kappa''^2} \qquad (25)$$

6)

Therefore:

$$\boxed{\kappa'' = \frac{mc}{\hbar}} \quad -(26)$$

The London equation of superconductivity and the Meissner effect are Beltrami equations of the type:

$$\nabla \times \underline{A} = i\kappa'' \underline{A} \quad -(27)$$

$$\nabla \times \underline{B} = i\kappa'' \underline{B} \quad -(28)$$

i.e.

$$\nabla^2 \underline{A} = \kappa''^2 \underline{A} \quad -(29)$$

$$\nabla^2 \underline{B} = \kappa''^2 \underline{B}. \quad -(30)$$

Using:

$$\underline{\nabla} \times \underline{B} = k\underline{B} = k\,\underline{\nabla} \times \underline{A} = k^2 \underline{A} \quad -(31)$$

the London equation of superconductivity is obtained:

$$\underline{J} = \underline{\nabla} \times \underline{B} = -\kappa''^2 \underline{A} \quad -(32)$$

The electric field from the London equation is zero:

$$\underline{E} = -\frac{\partial \underline{A}}{\partial t} = \underline{0} \quad -(33)$$

7) By Ohm's Law, the resistance of the conducting medium vanishes, and the medium is a superconductor. In a static magnetic assembly, there is no electric field, and the "medium" is spacetime itself. In this analogy, the spacetime can be regarded as a "superconductor" that excludes electric fields. It is a useful thought experiment to think of spacetime in this way.

The Beltrami analysis is a special case of eqn. (1). Taking the curl of both sides:

$$\langle \underline{\nabla} \times (\underline{\nabla} \times \underline{q}^a) \rangle = \langle \underline{\nabla} \times (\underline{\omega}^a{}_b \times \underline{q}^b) \rangle \quad -(34)$$

If it is assumed that:

$$\underline{\nabla} \cdot \underline{q}^a = 0 \quad -(35)$$

then:

$$\langle \nabla^2 \underline{q}^a \rangle = - \langle \underline{\nabla} \times (\underline{\omega}^a{}_b \times \underline{q}^b) \rangle \quad -(36)$$

Now use the vector identity:

8)

$$\underline{\nabla} \times \left(\underline{\omega}^a{}_b \times \underline{q}^b \right)$$
$$= \underline{\omega}^a{}_b \left(\underline{\nabla} \cdot \underline{q}^b \right) - \left(\underline{\omega}^a{}_b \cdot \underline{\nabla} \right) \underline{q}^b - \left(\underline{\nabla} \cdot \underline{\omega}^a{}_b \right) \underline{q}^b + \left(\underline{q}^b \cdot \underline{\nabla} \right) \underline{\omega}^a{}_b \quad -(37)$$

Further insight can be obtained into the balance condition as in paper 60, eqns. (65) ff. Following the arguments leading to eq. (67) of paper 60, the balance condition specializes to:

$$\left\langle \frac{\partial}{\partial z} \left(\frac{\partial q_x}{\partial x} \right) - \omega_z \frac{\partial q_x}{\partial x} - \left(\frac{\partial \omega_z}{\partial x} \right) q_x \right\rangle = 0 \quad -(38)$$

if only q_x is non-zero (an assumption made for simplicity.

Spin Connection Resonance (SCR)

If the balance condition is broken then eq. (38) may be developed into an SCR equation, e.g. eq. (67) of paper 60.

9) Wave Equation from Eq. (37)

If it is assumed that:

$$\underline{\nabla} \cdot \underline{q}^{b} = \underline{\omega}^{a}{}_{b} \cdot \underline{\nabla} = q^{b} \cdot \underline{\nabla} = 0 \quad (39)$$

then eq. (37) simplifies to:

$$\langle \nabla^{2} \underline{q}^{a} \rangle = \langle (\underline{\nabla} \cdot \underline{\omega}^{a}{}_{b}) \underline{q}^{b} \rangle \quad (40)$$

which is the generalization of eq. (22).

One of the key points here is as soon as the balance condition is broken, spin connection resonance can be used. So the inventor bring the assembly into SCR by trial and error.

In the balance condition, analogies can be drawn between spacetime, hydrodynamics, Beltrami fields, the Helmholtz equation, and superconductivity.