

Notes 74(1): Origin of Force, Torque and Energy in Magnetostatics

In generally covariant unified field theory the magnetic flux density in Tesla is defined by:

$$\underline{B}^a = \underline{\nabla} \times \underline{A}^a - \underline{\omega}^a{}_b \times \underline{A}^b \quad (1)$$

where  $\underline{A}^a$  is the vector potential and  $\underline{\omega}^a{}_b$  the spin connection vector. The magnetic flux density may be expressed as an angular momentum  $\underline{L}^a$  by:

$$\underline{B}^a = \left( \frac{\mu_0 e}{4\pi M r^3} \right) \underline{L}^a \quad (2)$$

where  $\mu_0$  is the permeability of the vacuum,  $e$  is the charge,  $M$  is the mass and  $\frac{4\pi r^3}{3}$  is a volume for a spherical symmetry.

In the standard model:

$$\underline{B} = \underline{\nabla} \times \underline{A} \quad (3)$$

and the spin connection is missing entirely.

Given the existence of a net magnetic dipole moment (permanent magnet):

$$\langle \underline{m} \rangle \neq \underline{0} \quad (4)$$

The spin connection generates the following linear force, torque and energy (Jackson, 3rd. ed.):

$$\underline{F}^a = \underline{\nabla} \left( \langle \underline{m} \rangle \cdot \underline{\omega}^a{}_b \times \underline{A}^b \right) \quad - (5)$$

$$\underline{T}^a = - \langle \underline{m} \rangle \times \left( \underline{\omega}^a{}_b \times \underline{A}^b \right) \quad - (6)$$

$$\underline{E}^a = \langle \underline{m} \rangle \cdot \underline{\omega}^a{}_b \times \underline{A}^b \quad - (7)$$

These quantities are missing from the standard model, which gives:

$$\underline{F} = \underline{\nabla} \left( \langle \underline{m} \rangle \cdot \underline{\nabla} \times \underline{A} \right) \quad - (8)$$

$$\underline{T} = \langle \underline{m} \rangle \times \left( \underline{\nabla} \times \underline{A} \right) \quad - (9)$$

$$\underline{E} = - \langle \underline{m} \rangle \cdot \underline{\nabla} \times \underline{A} \quad - (10)$$

The ECE theory gives the terms (8) to (10), and in addition, the terms (5) to (7). In the standard model, the magnetic field always obeys the law:

$$\underline{\nabla} \cdot \underline{B} = 0 \quad - (11)$$

and for magnetostatics the Ampère Law:

$$\underline{\nabla} \times \underline{B} = \mu_0 \underline{J} \quad - (12)$$

In ECE theory the corresponding laws are

$$\underline{\nabla} \cdot \underline{B}^a = \mu_0 \underline{j}^a \quad - (13)$$

$$\underline{\nabla} \times \underline{B}^a = \mu_0 \underline{J}^a \quad - (14)$$

The torque on a magnetic material in the standard model is therefore governed by:

$$\left. \begin{aligned} \underline{\nabla} \cdot \underline{B} &= 0 \\ \underline{\nabla} \times \underline{B} &= \mu_0 \underline{J} \\ \underline{T} &= \langle \underline{m} \rangle \times \underline{B} \end{aligned} \right\} - (15)$$

In ECE theory the torque is governed by:

$$\left. \begin{aligned} \underline{\nabla} \cdot \underline{B}^a &= \mu_0 \underline{j}^a \\ \underline{\nabla} \times \underline{B}^a &= \mu_0 \underline{J}^a \\ \underline{T}^a &= \langle \underline{m} \rangle \times \underline{B}^a \end{aligned} \right\} - (16)$$

From eq. (2), the torque in ECE theory is:

$$\underline{T}^a = \left( \frac{\mu_0 e}{m \gamma} \right) \langle \underline{m} \rangle \times \underline{L}^a \quad - (17)$$

where:

$$\underline{L}^a = \left( \frac{m \gamma}{\mu_0 e} \right) \left( \underline{\nabla} \times \underline{A}^a - \underline{\omega}^a \times \underline{A}^a \right) \quad - (18)$$

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i.e. :

$$\underline{L}^a = \left( \frac{mVA^{(0)}}{\mu_0 e} \right) \left( \underline{v} \times \underline{q}^a - \underline{\omega}^a{}_b \times \underline{q}^b \right)$$

where  $\underline{q}^a$  is the vector part of the Cartan tetrad:

$$q^a_\mu = (q^a_0, -\underline{q}^a). \quad (20)$$

The tetrad defines the Cartan torsion:

$$T^a = d \wedge q^a + \omega^a{}_b \wedge q^b \quad (21)$$

is the notation of differential geometry.

So the origin of the torque defined in eq (16) is the Cartan torsion of spacetime.

In the standard model the vector potential  $\underline{A}$  is used as a mathematical quantity with no relation to the Cartan torsion.

Therefore in ECE theory the torque on a magnetic material of net magnetic dipole moment  $\langle m \rangle$  is:

5)

$$\underline{T}^a = A^{(0)} \langle \underline{m} \rangle \times (\underline{\omega} \times \underline{q}^a - \underline{\omega}^a \times \underline{q}^b) \quad (22)$$

### The Rotating Magnet

If:  $\langle \underline{T}^a \rangle \neq \underline{0} \quad (23)$

The magnet will rotate, because the non-zero net torque produces an angular momentum. This rotating magnet is the basis for a magnetic rotor. In order to give the result (23) the average of the right hand side of eq. (22) must be non-zero. If eq. (22) is applied at the molecular level, it produces a Lagrangian function and anisotropy, new cross correlation functions in the lab. frame and moving frame, but the sample as a whole does not rotate. In a molecular ensemble of moving molecules:

$$\langle \underline{m} \rangle = \underline{0} \quad (24)$$

unless the magnetization  $M$  is non-zero

6) In a permanent magnet however there is a permanent and non-zero  $\langle \underline{m} \rangle$ . So in order to produce a non-zero  $\underline{I}^a$  from eq. (22):

$$A^{(0)} \langle \underline{\nabla} \times \underline{q}^a - \underline{\omega}^a{}_b \times \underline{q}^b \rangle \neq 0. \quad (25)$$

The presence of  $A^{(0)}$  comes from the presence of a permanent magnetization, i.e. the use of a permanent magnet. Thus:

$$\langle \underline{\nabla} \times \underline{q}^a \rangle \neq \langle \underline{\omega}^a{}_b \times \underline{q}^b \rangle$$

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This is the condition needed for a magnetic motor. This condition is equivalent to:

$$\langle \underline{\nabla} \times \underline{A}^a \rangle \neq \langle \underline{\omega}^a{}_b \times \underline{A}^b \rangle \quad (27)$$

(Clearly, this condition cannot exist in the standard model because the concepts of  $\underline{q}^a$  and  $\underline{\omega}^a{}_b$  are missing.)