APPLICATIONS OF THE ECE INVARIANCE PRINCIPLE

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ABSTRACT

The invariance principle of the Einstein Cartan Evans (ECE) unified field theory states that the tetrad postulate is invariant under the general coordinate transformation. It is shown that the invariance principle implies a global component in the definition of phase in physics. This global component is the origin of several well known effects such as the class of Aharonov Bohm effects. The phase in optics and electrodynamics must be defined in such a way as to conserve parity inversion symmetry, and it is shown that the correct definition of phase in this context is in terms of the global component from the invariance principle and the fundamental ECE spin field. The global component is always related to a local component of phase through a Stokes Theorem with covariant derivative. The global component of phase in quantum mechanics implies the existence of a global action or angular momentum. The latter is the global phase multiplied by the reduced Planck constant. Quantum entanglement and one photon interferometric effects are explained with the global action. The latter is derived from general relativity.

Keywords: Invariance principle, ECE theory, covariant Stokes Theorem, global action and angular momentum.

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1. INTRODUCTION

The tetrad postulate is fundamental to standard Cartan (or differential) geometry {1-11}. Without it, Cartan geometry does not reduce correctly to Riemann geometry, so without it, Cartan geometry would be meaningless, a reduction to absurdity. As is well known {1}, Cartan geometry is self-consistently defined by its two structure equations and two Bianchi identities, together with the tetrad postulate. The tetrad postulate is fundamental because it is a statement of the fact that a complete vector field must be independent of its components and basis elements. For example, a vector field in three dimensional Euclidean geometry can be expressed either in terms of Cartesian coordinates:

$$ \underline{V} = V_x \, \underline{i} + V_y \, \underline{j} + V_z \, \underline{k} \qquad -(1) $$

or spherical polar coordinates:

$$ \underline{V} = V_\theta \, \underline{e}_\theta + V_\phi \, \underline{e}_\phi + V_r \, \underline{e}_r \qquad -(2) $$

but is the same vector field. Here V_x, V_y, V_z are the Cartesian vector components, i, j and $\underline{k}$ are the Cartesian unit vectors (the basis elements). Similarly, $V_\theta, V_\phi, \overline{V_r}$ are the spherical polar vector components, and $\underline{e}_\theta$, $\underline{e}_\phi$ and $\underline{e}_r$ are the spherical polar unit vectors {12}. The tetrad postulate is the result of extending this fact to more than three dimensions and to a more general base manifold {1}. It states that the covariant derivative of the tetrad vanishes:

$$ D_\mu \, q^a_{\;\nu} = 0. \qquad -(3) $$

The tetrad is a rank two mixed index tensor, one index a represents the tangent space-time, a Minkowski space-time, and the other index μ represents the base manifold. The tetrad postulate is true for any number of dimensions, but in ECE theory and general relativity there

are four dimensions, one time dimension and three space dimensions.

It follows {11} from Eq. (3) that the tetrad postulate is invariant under any type of coordinate transformation:

$$D_\mu V_\nu^a = \left(D_\mu V_\nu^a \right)' = 0 \quad -(4)$$

and this is the invariance principle of ECE theory. In other words the tetrad postulate is an invariant of general relativity and Cartan geometry. Invariants play an important role in relativistic physics, examples being the two Casimir invariants of the Poincaré group of special relativity, the mass and spin invariants {13}. In Section 2 it is shown that the invariance principle leads to the existence of a global component of phase in physics. This global component is always related to a local component through a Stokes theorem with the correctly defined covariant derivative and correctly defined path of integration. So the local and global components both derive from the field theory of general relativity. Action at a distance is not needed, and the constancy of the speed of light does not conflict with global effects in physics. The distinction between local and global is therefore artificial and redundant. The constancy of the speed of light has been verified experimentally in special relativity to an accuracy of one part in nearly thirty orders of magnitude. In general relativity the constancy of the speed of light has been verified experimentally by NASA Cassini to one part in about 100,000. The photon mass is therefore also known to this accuracy by implication, although the numerical value of the photon mass is known only through the experimental fact that it must be less than a certain number of kilograms. These data therefore show that the field theory of relativity is the most accurate theory by far in physics. ECE theory {2-11} is an extension of the original Einstein Hilbert (EH) field theory of general relativity (1916) to include the Cartan torsion. In so doing the EH theory of gravitation is

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extended to a generally covariant unified field theory of all the fundamental force fields of physics: gravitation, electromagnetism, weak and strong fields, and matter fields. ECE therefore introduces a new physics wherein the interaction between the fundamental fields can be understood in causal terms, covariantly and objectively as demanded by the Einsteinian and Baconian principles of natural philosophy. ECE also unifies quantum mechanics and relativity by using the tetrad postulate to construct the fundamental wave equation of physics, the ECE wave equation {2-11}. The latter is based on the ECE Lemma which is straightforwardly derived from the tetrad postulate. It has been shown that the well known wave equations of physics, such as the Dirac, Schrödinger and Proca equations, are limits of the ECE wave equation. The global component of phase in optics and

electrodynamics is given in terms of the fundamental ECE spin field $\underline{B}^{(3)}$ {2-11}. The global component of phase, α, implies in quantum mechanics the existence of a global action or angular momentum, $\hbar \alpha$, where $\hbar$ is the reduced Planck constant. The phase α is related to the spin connection of the spinning spacetime that defines electromagnetism in ECE theory. A simple example is given of the relation between α and the spin connection for the potential plane wave in electrodynamics.

In Section 3 some discussion is given of the role of α in the Aharonov Bohm effects, quantum entanglement, the topological phases, and one photon Young interferometric effects.

2. THE GLOBAL AND LOCAL PHASES IN PHYSICS.

The invariance principle (4) means that the tetrad must transform as follows under arbitrary change of coordinates:

$$ q^a_{\mu'} \rightarrow e^{i\alpha} q^a_{\mu} \quad -(5) $$

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where α must be independent of space and time, i.e. independent of

$$x^\mu = (ct, X, Y, Z). \quad -(6)$$

Since there is no preferred frame of reference in relativity theory the tetrad takes the form (5) in general: the global phase α is in general non-zero. It is termed "global" because in the received description, "global" is applied to an object that is not dependent on x^μ, "local" to an object that depends on x^μ. It is shown in this section that these descriptions are arbitrary and redundant, because the global and local phases are always linked by a Stokes theorem with covariant derivatives. Thus, global effects are implied by the local field theory of relativity, and there is no contradiction between relativity and quantum mechanics, no contradiction between local and global effects in relativity. These deductions are shown to follow from the invariance principle of geometry. In the received description, global effects are also described as "non-local" and the latter may be described in terms of α. In special relativity this is not possible, because the spin connection is missing, the space-time of special relativity is the Minkowski space-time, and this is flat and static.

The invariance principle implies that the lagrangian density ($\mathcal{L}$) is invariant under the general coordinate transformation because {2-11}:

$$\mathcal{L} = c^2 T + D_\mu q_\nu^a D^\mu q^\nu_a \quad -(7)$$

where T is the index reduced canonical energy momentum defined by:

$$R = -kT \quad -(8)$$

where R defined by the ECE Lemma and where k is the Einstein constant. The Euler Lagrange equation:

$$\frac{\partial \mathcal{L}}{\partial q^{\tilde{}}_{Va}} = D^\mu \left(\frac{\partial \mathcal{L}}{\partial (D^\mu q^{\tilde{}}_{Va})} \right) \quad - (9)$$

then correctly gives the covariant derivative of the tetrad postulate as the following identity:

$$D^\mu \left(D_\mu q^a_{\tilde{}} \right) : = 0. \quad - (10)$$

From Eq (10) the ECE Lemma is obtained straightforwardly {2-11}:

$$\Box q^a_\mu = R q^a_\mu \quad - (11)$$

where R is a scalar curvature defined by:

$$R := q^\lambda_a \partial^\mu \left(\Gamma^{\tilde{}}_{\mu\lambda} q^a_{\tilde{}} - \omega^a_{\mu b} q^b_\lambda \right). - (12)$$

Here $\Gamma^{\tilde{}}_{\mu\lambda}$ is the generally defined gamma connection of Riemann geometry and $\omega^a_{\mu b}$ is the spin connection of Cartan geometry. The gamma connection is the Christoffel connection if and only if it is symmetric in its lower two indices:

$$\Gamma^{\tilde{}}_{\mu\lambda} = \Gamma^{\tilde{}}_{\lambda\mu}. \quad - (13)$$

In this limit the torsion tensor vanishes:

$$T^{\tilde{}}_{\mu\lambda} = \Gamma^{\tilde{}}_{\mu\lambda} - \Gamma^{\tilde{}}_{\lambda\mu} = 0 \quad - (14)$$

and this is the limit that defines the EH theory. More generally the Riemann torsion tensor and the Cartan torsion form are non-zero, and this defines ECE theory.

The action is defined {2-12} as:

$$S = \frac{1}{c} \int \mathcal{L} d^4 x \quad - (15)$$

in S.I. units, where c is the speed of light, the universal constant of the field theory of relativity (Einstein, 1905). Under the general coordinate transformation the action is changed in general to:

$$S \rightarrow S' = \frac{1}{c} \int \mathcal{L} \, d^4x' \quad -(16)$$

It has been shown {2-11} that the tetrad propagates in general through the equation:

$$q_\mu^a(x^\mu) = exp\left(i \frac{S}{\hbar}(x^\mu)\right) q_\mu^a(o) - (17)$$

which is an equation that combines the Fermat and Hamilton principles of least time and action respectively. These principles are at the root of quantum mechanics as is well known. Under the general coordinate transformation therefore, Eq. (17) is changed to:

$$q_{\mu'}^a(x^{\mu'}) = exp\left(i\left(\frac{S(x^\mu)}{\hbar} + \alpha\right)\right) q_\mu^a(o) - (18)$$

and since there is no preferred frame of reference the tetrad is in general:

$$q_\mu^a = exp\left(i\left(\frac{S(x^\mu)}{\hbar} + \alpha\right)\right) q_\mu^a(o). - (19)$$

Under the transformation (18) the lagrangian density (7) is always invariant. Comparing Eqs. (16) and (19) it is seen that the integral over the four volume d^4x must be invariant, although its time and space components are individually changed by the general coordinate transformation. Therefore the action is invariant under the general coordinate transformation when the lagrangian density is defined by Eq. (7). The latter gives the correct identity (10) using the Euler Lagrange equation (9).

The phase defined in Eq. (19) contains a local part $\frac{S(x^\mu)}{\hbar}$ that is a function of

x^{μ}, and a global part α that is not a function of x^{μ}. The action is invariant under the general coordinate transformation, as we have seen, so the local part $\frac{S(x^{\mu})}{\hbar}$ is also invariant. The global part α changes under the general coordinate transformation. These are the general properties of phase of any type in physics, for example the electrodynamic and fermionic phases and the topological phases and in the wave-functions of quantum mechanics. They are also present in the gravitational, weak and strong fields. The electromagnetic potential plane wave propagating in Z, for example, must be defined in general by:

$$\underline{A}^{(1)} = \frac{\underline{A}^{(0)}}{\sqrt{2}}\left(\underline{i} - i\underline{j}\right)\exp\left(i\left(\omega t - \kappa Z + \alpha\right)\right) \qquad -(20)$$

where $cA^{(0)}$ has the units of volts in S.I., and where ω is the angular velocity of the plane wave at instant t, and where κ is the wave-number magnitude at Z. For a plane wave of this type the general coordinate transformation reduces to the Lorentz transformation under which:

$$\omega t - \kappa Z = \omega' t' - \kappa' Z' \qquad -(21)$$

as is well known. However α changes as we have argued. ECE theory has shown {2-11} that this type of plane wave is an example of a tetrad defined by the rotation of a frame with respect to a frame that defines the handedness of the plane wave (i.e. its left or right circular polarization). In the generally covariant ECE unified field theory all potential fields are tetrads. The electromagnetic field itself is defined from the tetrad potential field through the first structure relation of Cartan geometry as follows:

$$F^a = d \wedge A^a + \omega^a{}_b \wedge A^b \qquad -(22)$$

So the electromagnetic field is the Cartan torsion within the factor $A^{(0)}$:

$$F^a_{\mu\nu} = A^{(0)} T^a_{\mu\nu} \quad - \quad (23)$$

and the electromagnetic potential field is the Cartan tetrad through the same factor $A^{(0)}$:

$$A^a_\mu = A^{(0)} q^a_\mu \quad - \quad (24)$$

For pure rotational motion {2-11} the spin connection is dual as follows to the tetrad:

$$\omega^a_{\mu b} = -\frac{\kappa}{2} \epsilon^a_{bc} q^c_\mu \quad - \quad (25)$$

so it is seen that the spin connection must be a function of α:

$$\omega^a_{\mu b} = \omega^a_{\mu b}(\alpha). \quad - \quad (26)$$

This is true for all fields, the spin connection has a global component that depends on α as well as a local component.

It has been shown {2-11} that the Aharonov Bohm effects, quantum entanglement, one photon Young interferometry, the Sagnac effect (ring laser gyro), the Faraday disk effect and similar are all due to the spin connection of ECE theory, so they are all related to the global α which derives from the invariance principle. Under the general coordinate transformation the spin connection changes as follows:

$$\omega^a_{\mu b} = e^{i\alpha} \omega^a_{\mu b}. \quad - \quad (27)$$

The archetypical quantum entanglement effect for example is observed when two particles are well separated in distance and still appear to be correlated in their dynamics (for example

spins). It is inferred that this correlation takes place through α, which does not depend on distance and time. Similar effects of α are seen in one photon Young interferometry. In this case, interferograms are observed even though only one photon is present and can pass through only one aperture out of two. The photon in ECE theory is defined by the ECE wave equation:

$$\left(\Box + kT \right) A_{\mu}^{a} = 0 \qquad - (28)$$

where A_{μ}^{a} is the wave-function and potential field. When the electromagnetic field is free of the influence of any other field, for example a matter field or gravitational field, Eq. (28) reduces to:

$$\left(\Box + \frac{m^2 c^2}{\hbar^2} \right) A_{\mu}^{a} = 0 \qquad - (29)$$

because

$$kT \rightarrow \frac{m^2 c^2}{\hbar^2} \qquad - (30)$$

for a free electromagnetic field. Eq. (29) is the well known Proca equation {2-11} for each polarization index a. In Eq. (29) the particulate part of the photon is defined by the eigen-value, $\frac{m^2 c^2}{\hbar^2}$, which contains the photon mass, and the wave or undulatory part of the photon by the eigen-function A_{μ}^{a}. As seen from the example given in Eq. (20), the wave part contains the global α and the particulate "core" of the photon does not. Therefore the particulate part passes through the opening of one aperture of the Young interferometer, while the wave part sets up the interferogram by going through both apertures. This means that both particle and wave are simultaneously observable, contradicting the Bohr Heisenberg indeterminacy experimentally. The distinction between particle and wave becomes arbitrary and redundant. It is this artificial distinction that has caused so much debate in physics over

the centuries. The wave part, as we have argued, always contains a local and global component. These will be defined more precisely later in this section, but firstly some remarks are made on the gauge principle.

The standard model uses the gauge principle, which asserts that the action is invariant under the local gauge transformation of a field ψ as follows:

$$\psi' = \sigma(x^\mu)\psi. \qquad - (31)$$

This is usually applied as a principle of special relativity {13} and

$$\partial_\mu \psi' = \sigma \partial_\mu \psi + (\partial_\mu \sigma)\psi \qquad - (32)$$

is not covariant because σ is a function of x^μ. Gauge theory then introduces a type of covariant derivative in Minkowski space-time through which:

$$D'_\mu \psi' = \sigma D_\mu \psi. \qquad - (33)$$

In electrodynamics for example it is usually assumed that the gauge theory is U(1) invariant and:

$$D'_\mu = \partial_\mu - ig A'_\mu, \qquad - (34)$$

$$D_\mu = \partial_\mu - ig A_\mu. \qquad - (35)$$

Thus:

$$A'_\mu = \sigma A_\mu \sigma^{-1} - \frac{i}{g}(\partial_\mu \sigma)\sigma^{-1} - (36)$$

in accordance with the Maxwell-Heaviside (MH) field equations. This is the way that the potential field is introduced in the standard model and it has been shown {2-11} that the method is fraught with shortcomings and contradictions which are not present in ECE theory. For example, the standard model's gauge principle cannot produce the Proca equation

because the required action is not invariant under the gauge transformation. This conflicts with the fact that NASA Cassini has verified the existence of photon mass to great accuracy. Even though the photon mass is known only to be less than a certain value, its existence has been verified to this accuracy because NASA Cassini measured light deflection due to gravitation in the solar system to this accuracy. This deflection does not occur without photon mass, and is caused by the gravitational attraction of the photon of mass m and sun of mass M in EH theory. Another well known shortcoming of the gauge theory and standard model {13} of electromagnetism is that the potential field is not manifestly or fully covariant. Only two of its four components are considered to be physical in canonical quantization - the transverse components by the Gupta Bleuler method {13 }. This problem is again caused by the neglect of photon mass, a "massless particle" (the photon) in special relativity only has two states of polarization, and these are asserted to be the transverse parts. In ECE theory there are four components of the potential field as required by manifest covariance, and the photon mass is identically non-zero as required by NASA Cassini. Therefore in ECE there are no problems {2-11} with canonical or second quantization. Therefore the gauge principle should be replaced by the more fundamental invariance principle throughout physics.

It has been argued that the phase contains a local and global part, for example as follows:

$$\phi = \omega t - \underline{\kappa} \cdot \underline{r} + \alpha. \quad - (37)$$

The complete phase factor is therefore:

$$\gamma = \exp\left(i\left(\omega t - \underline{\kappa} \cdot \underline{r} + \alpha\right)\right). \quad - (38)$$

In quantum mechanics the action is therefore:

$$S = \hbar\left(\omega t - \underline{\kappa} \cdot \underline{r} + \alpha\right) \quad - (39)$$