71(5) : Review of Aharonov Bohm Effects in ECE Theory

In chapter 10 of vol. 1 it was argued that the phase transformation:

$$\phi \rightarrow e^{i\alpha}\phi \quad -(1)$$

generates the Dirac Wu Yang phase from the conventional phase $\phi$ of the MH field theory. Since $\alpha$ is arbitrary, this transformation is not gauge invariant, contrary to the usual assertion that MH is a U(1) gauge invariant theory. It was then proven that the MH theory is incorrectly applied to the AB effect in conventional texts. This was proven by considering the Stokes Theorem:

$$\exp\left(ig\oint_{\delta S} A\right) = \exp\left(ig\int_S d\wedge A\right) \quad -(2)$$

which shows that the AB effect cannot be due to a gauge transformation:

$$A \rightarrow A + d\chi \quad -(3)$$

because:

$$\oint_{\delta S} d\chi = \int_S d\wedge d\chi = 0 \quad -(4)$$

by the Poincaré Lemma:

$$d\wedge d := 0. \quad -(5)$$

The correct description of the AB effect was given through the non-Abelian Stokes Theorem:

$$\oint_{DS} A = \int_S D\wedge A \quad -(6)$$

where:

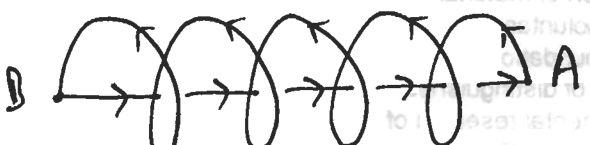
$$F = D \wedge A \qquad (7)$$

$$= d \wedge A + \omega \wedge A$$

It was concluded that the electromagnetic phase in generally covariant electrodynamics takes the form:

$$\Phi = \exp\left( ig \oint_{DS} A^{(3)} \right) = \exp\left( ig \int_S B^{(3)} \right) \qquad (8)$$

This equation was interpreted as integration around a helix:




— (9)

The only non-zero contribution is from B to A.

This general phase theory was then applied to the Sagnac and Aharonov Bohm effects. The latter was described by:

$$\exp\left( ig \int \underline{B}^{(3)} \cdot \underline{k}\, dAr \right) = \exp\left( ig \oint \underline{A}^{(3)} \cdot d\underline{r} \right)$$

$$= \exp\left( g^2 \int \underline{A}^{(1)} \times \underline{A}^{(2)} \cdot \underline{k}\, dAr \right) \qquad (10)$$

where:

$$\underline{B}^{(3)*} = -ig\, \underline{A}^{(1)} \times \underline{A}^{(2)}. \qquad (11)$$

3)

In chapter 24 of vol. 1 this theory was developed by considering:

$$\alpha^a = \int_S D \wedge F^a = \oint F^a + \int_S \omega^a{}_b \wedge F^b \quad -(12)$$

and the magnetic flux:

$$\Phi^a = \oint A^a + \int_S \omega^a{}_b \wedge A^b \quad -(13)$$

$$= \int_S F^a \quad -(14)$$

The AB effects are observed when, experimentally:

$$\oint A^a = 0. \quad -(15)$$

For example, in the region enclosed by the electron beams in the Chambers experiment, eq. (15) is true outside the iron whisker. The magnetic flux observed in the experiment (the AB effect) is ~~therefore~~:

$$\Phi^a = A^{(0)} \int_S \omega^a{}_b \wedge q^b. \quad -(16)$$

this is generated by a phase:

$$\boxed{\phi^a = \frac{e}{\hbar} \Phi^a} \quad -(17)$$

In the Maxwell Heaviside theory, if:

$$\oint A = 0, \quad -(18)$$

then:

$$\Phi = \int_S F = \oint A = 0 \quad -(19)$$

and

$$\boxed{\phi = \frac{e}{\hbar}\,\Phi = 0.} \quad -(20)$$

So there is no AB effect in MH theory.

For each $a$ in eq. (17):

$$\boxed{\phi^a = \frac{e}{\hbar} A^{(0)} \int \omega^a{}_b \wedge q^b \neq 0} \quad -(\cancel{18}\ 21)$$

in ECE theory. The phase factor is therefore:

$$\gamma^a = \exp\left(i\phi^a\right). \quad -(\cancel{19}\ 22)$$

In paper 71, it is shown that such a phase factor arises from the general coordinate transformation of the ECE wave equation. In notes 71(1) to 71(3) this transformation was shown to be:

$$(\square + kT)\, A^a_\mu \rightarrow (\square + kT)\, A^{a\prime}_\mu \quad -(23)$$

i.e.

$$A^{a\prime}_\mu = A^a_\mu \exp\left(i\alpha\right). \quad -(24)$$

For each $a$ we may now identify $\phi$ of eq. (22) with $\alpha$ of eq. (24).

5) The AB effects were again developed in chapter 2 of volume 2, where the Chambers experiment in the complex circular basis was described by the magnetic flux:

$$\Phi^{(3)*} = \int_S F^{(3)*} = \oint A^{(3)*} - i\frac{e}{\hbar}\int_S A^{(1)} \wedge A^{(2)} \quad -(25)$$

Here, $F^{(3)}$ and $A^{(3)}$ are confined to the iron whisker whereas $A^{(1)} \wedge A^{(2)}$ exists outside the iron whisker. They are generated by the rotation of spacetime itself. In paper 71 we now understand that this rotation is a coordinate transformation of the ECE wave equation or Lemma.

It is also understood that the AB effect is an example of a whole class of phenomena that are due to spacetime rotation, this class includes the AB, Sagnac and Faraday disk effects. The basic principle at work in this class of effects is:

$$\boxed{D_\nu q^a_\mu = (D_\nu q^a_\mu)' = 0} \quad -(26)$$

This is the invariance of the tetrad postulate, and is more fundamental than the gauge principle.

6) The Sagnac Effect.

This was developed in chapter 1 of volume 3 as the phase transformation:

$$\omega_1 t \rightarrow (\omega_1 \pm \Omega) t \qquad -(27)$$

The Sagnac effect is the time delay:

$$\Delta t = 2\pi\left(\frac{1}{\omega_1 - \Omega} - \frac{1}{\omega_1 + \Omega}\right) \qquad -(28)$$

The corresponding phase factor transformation is:

$$\exp(i\omega t) \rightarrow \exp(\pm i\Omega t)\exp(i\omega t) \qquad -(29)$$

and this is again due to the general coordinate transformation (23). The latter is a consequence of the invariance principle (26).

In quantum mechanics, entanglement and Young interferometric effects are due to the same principle. The Sagnac effect is again due to the rotation of spacetime itself, thus proving ECE theory in one way out of many.