1) 70(4): Field Unification, Fundamental Elements and Minimal Prescription.

In this part the meaning of the minimal prescription is ECE will be carefully defined. It is first noted that the chirality and spin vectors defined in notes 70(2) can be defined in terms of the fundamental elements ξ_1 and ξ_2 defined in notes 70(3) as follows:

$$V_L^{(1)} = \frac{1}{\sqrt{2}}\left(\begin{bmatrix} 0 \\ \xi_1 \\ 0 \end{bmatrix} - i\begin{bmatrix} 0 \\ \xi_2 \\ 0 \end{bmatrix}\right) \quad -(1)$$

$$V^{u} = \left(\begin{bmatrix} 0 \\ \xi_1 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ \xi_2 \\ 0 \end{bmatrix}\right) e^{-i\phi} \quad -(2)$$

This means that the same elements:

$$\xi_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad \xi_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad -(3)$$

define both the electromagnetic and the fermion field.

The tetrad elements of the electromagnetic field can be put in SU(2) form by using the relation in chapter 6 of volume one:

$$R^2 = X^2 + Y^2 + Z^2 = \xi_1 \xi_1^* + \xi_2 \xi_2^* \quad -(4)$$

where ξ in eq. (4) denotes the fundamental spinor of SU(2). Thus:

$$\underline{q}^{(1)} \cdot \underline{q}^{(2)} + \underline{q}^{(2)} \cdot \underline{q}^{(1)} + \underline{q}^{(3)} \cdot \underline{q}^{(3)}$$

$$= q_1 q_1^* + q_2 q_2^* \quad -(5)$$

2) Eq. (5) gives the SU(2) rep on the RHS and the O(3) complex circular rep. on the LHS. For circular polarization:

$$\underline{q}^{(1)} = \frac{1}{\sqrt{2}} \left(\underline{i} - i\underline{j} \right) e^{i\phi} = \underline{q}^{(2)*} , \quad -(6)$$

$$\underline{q}^{(3)} = \underline{k} . \quad -(7)$$

Therefore:

$$q_1 q_1^* + q_2 q_2^* = 3 \quad -(8)$$

and a possible solution in SU(2) rep. is:

$$q_1 = \frac{3}{\sqrt{2}} e^{i\phi} , \quad q_2 = -\frac{3}{\sqrt{2}} i e^{i\phi} \quad -(9)$$

Geometrically, the e/m field has the same origin as the fermion field. Both can be put in SU(2) rep. space.

<u>Minimal Prescription</u> in <u>ECE Theory</u>

The ECE Lemma is:

$$\Box q^a_\mu = R q^a_\mu \quad -(10)$$

where:

$$\Box = \partial_\mu \partial^\mu = \gamma^\mu \gamma^\nu \partial_\mu \partial_\nu \quad -(11)$$

where γ^μ is the Dirac matrix. The d'Alembertian is the same in all spaces, flat or curved, and so are γ^μ and ∂^μ. The ECE Lemma

3) and wave equation were constructed carefully in such a way as to isolate the d'Alembertian $\Box$. This is needed in order to introduce the minimal prescription, which was used by Dirac to deduce the half-integral fermion spin as is well known. The minimal prescription describes the interaction of the fermion and the classical e/m field. If the latter is quantized, we must use quantum electrodynamics (see vol. 1, chapter 21).

Firstly, the quantum operator equivalence is introduced as follows:

$$p^\mu = i\hbar \partial^\mu \quad -(12)$$

where:

$$p^\mu = \left(\frac{E_n}{c}, \underline{p} \right), \quad \partial^\mu = \left(\frac{1}{c}\frac{\partial}{\partial t}, -\underline{\nabla} \right) \quad -(13)$$

Now define:

$$A_\mu := A_\mu^{(0)} + A_\mu^{(1)} + A_\mu^{(2)} + A_\mu^{(3)} \quad -(14)$$

so that all four states of polarization of A_μ^a are accounted for. The semi-classical minimal prescription in ECE theory is then defined as:

$$p_\mu \rightarrow p_\mu + eA_\mu \quad -(15)$$

$$p^\mu \rightarrow p^\mu + eA^\mu . \quad -(16)$$

This means that:

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$$\partial_\mu \to \partial_\mu - i\frac{e}{\hbar}A_\mu \quad -(17)$$

$$\partial^\mu \to \partial^\mu - i\frac{e}{\hbar}A^\mu \quad -(18)$$

and :

$$\Box = \partial_\mu \partial^\mu \to \left(\partial_\mu - i\frac{e}{\hbar}A_\mu\right)\left(\partial^\mu - i\frac{e}{\hbar}A^\mu\right) \quad -(19)$$

i.e.

$$\Box' := \Box - i\frac{e}{\hbar}\left(A_\mu \partial^\mu + A^\mu \partial_\mu\right) - \frac{e^2}{\hbar^2}A_\mu A^\mu \quad -(20)$$

The wave equation that describes the interaction of the Dirac spinor ϕ and A_μ is therefore :

$$(\Box' + kT)\phi = 0 \quad -(21)$$

In the absence of gravitation this becomes :

$$\left(\Box' + \left(\frac{m_e c}{\hbar}\right)^2\right)\phi = 0 \quad -(22)$$

Eq (22) produces the familiar half-integral spin effects such as the Stern Gerlach experiment, the Zeeman effect, ESR, NMR and MRI, and also RFR.

So the overall effect is :

$$\boxed{\Box \to \Box'} \quad -(23)$$

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Any type of field interaction can be described in this way. If for example we wish to investigate the effect of gravitation on interacting e/m and fermion fields equation (21) must be used. Both the fermion field and the electromagnetic field are affected by gravitation. The latter effect is governed by the field equations - the two Cartan structure equations and the two Bianchi identities. For the fermion field:

$$T^a = d \wedge q^a + \omega^a{}_b \wedge q^b \quad -(24)$$

$$R^a{}_b = d \wedge \omega^a{}_b + \omega^a{}_b \wedge \omega^b{}_c \quad -(25)$$

$$D \wedge T^a := R^a{}_b \wedge q^b \quad -(26)$$

$$D \wedge R^a{}_b := 0. \quad -(27)$$

Eqs. (24) and (25) are the two Cartan structure equations, and eqs. (26) and (27) are the two Bianchi identities. The fermion field is q^a in SU(2) rep. space. The effect of gravitation on q^a is given via the Riemann form using eqs. (24) and (26). All eqs. (24) to (27) must be written in SU(2) rep. space.

To find the effect of gravitation on the e/m field we use the ECE Ansatz:

$$A^a = A^{(0)} q^a, \quad -(28)$$

$$F^a = A^{(0)} T^a. \quad -(29)$$

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Finally the effect of the e/m field on the fermion field is found using eq. (23). None of this can be done in the standard model.

If we go to the next stage, a fully quantum treatment, the e/m field is itself quantized by:

$$\left(\Box + kT \right) A_\mu^a = 0 \quad -(30)$$

From eqs. (15) and (16) and conservation of momentum, the photon momentum is changed by the electron momentum. If the electron momentum is increased after collision, the photon momentum is decreased as follows:

$$A_\mu \rightarrow A_\mu - \frac{1}{e} p_\mu \quad -(31)$$

so the sum of momenta before and after collision is the same, (see volume one, chapter 21). Thus equation (30) is changed to:

$$\left(\Box'' + kT \right) A_\mu^a = 0 \quad -(32)$$

where:

$$\Box'' = \Box + \frac{i}{\hbar} \left(p_\mu \partial^\mu + p^\mu \partial_\mu \right) - \frac{p_\mu p^\mu}{\hbar^2} \quad -(33)$$

If there is no gravitation present:

$$kT = \left(m_p c / \hbar \right)^2 \quad -(34)$$

where m_e is the photon mass.

Therefore the equations:

$$\left(\square' + \left(\frac{m_e c}{\hbar} \right)^2 \right) \phi = 0 \quad -(35)$$

and

$$\left(\square'' + \left(\frac{m_p c}{\hbar} \right)^2 \right) A^a_\mu = 0 \quad -(36)$$

must be solved simultaneously. The electron mass m_e is many orders of magnitude greater than the photon mass m_p. In most textbook treatments of, for example, the Compton effect the energy of the electron is represented classically and relativistically by:

$$E_n = (m^2 c^4 + p^2 c^2)^{1/2} \quad -(37)$$

and the photon is represented by:

$$E_n = \hbar\omega, \quad p = \hbar\omega / c \quad -(38)$$

In Feynman's q.e.d., exchange of a virtual photon is used. However, in general relativity (ECE), eqns. (35) and (36) must be solved simultaneously. This must be done with sufficient numerical precision to give the anomalous magnetic moment of the electron and the Lamb shift and Casimir effect.