1) 70(2): The Origin of Circular Polarization in Electromagnetism, ECE Theory.

Electromagnetism is described in ECE theory by the 4 x 4 tetrad q_μ^a defined by:

$$V^a = q^a_\mu V^\mu \quad -(1)$$

where V^a is the chirality column vector and V^μ is the spin column vector. In general:

$$a = (0), (1), (2), (3) \quad -(2)$$

and,

$$\mu = 0, X, Y, Z. \quad -(3)$$

It was discovered by Arago in 1811 that e/m is left (1) and right (2) circularly polarized. E/m also has a timelike ((0)) and longitudinal ((3)) sense of polarization. So, for example, the left circularly polarized e/m potential is:

$$\underline{A}^{(1)}_L = \frac{A^{(0)}}{\sqrt{2}} (\underline{i} - i\underline{j}) e^{i\phi} \quad -(4)$$

and the right c. p. e/m potential is:

$$\underline{A}^{(1)}_R = \frac{A^{(0)}}{\sqrt{2}} (\underline{i} + i\underline{j}) e^{i\phi} \quad -(5)$$

where ϕ is the electromagnetic phase.

2) The complex conjugates of eqs (4) and (5) are:

$$\underline{A}_L^{(2)} = \frac{A^{(0)}}{\sqrt{2}}\left(\underline{i} + i\underline{j}\right) e^{-i\phi} \quad -(6)$$

$$\underline{A}_R^{(2)} = \frac{A^{(0)}}{\sqrt{2}}\left(\underline{i} - i\underline{j}\right) e^{-i\phi} \quad -(7)$$

The complex circular basis is:

$$\underline{e}^{(1)} \times \underline{e}^{(2)} = i\underline{e}^{(3)*} \quad -(8)$$

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where:

$$\underline{e}^{(1)} = \frac{1}{\sqrt{2}}\left(\underline{i} - i\underline{j}\right) = \underline{e}^{(2)*} \quad -(9)$$

$$\underline{e}^{(3)} = \underline{k}. \quad -(10)$$

<u>Left Circular Polarization</u>

$$V_L^{(1)} = \frac{1}{\sqrt{2}}\begin{bmatrix} 0 \\ 1 \\ -i \\ 0 \end{bmatrix}, \quad V^\mu = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix} e^{-i\phi} \quad -(11)$$

Therefore:

$$q_{L\mu}^{(1)} = \frac{1}{\sqrt{2}}\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -i & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} e^{i\phi} \quad -(12)$$

so:

$$q_{LX}^{(1)} = \frac{1}{\sqrt{2}} e^{i\phi} \quad -(13)$$

$$q_{LY}^{(1)} = -\frac{i}{\sqrt{2}} e^{i\phi}. \quad -(14)$$

and:

$$\underline{q}_L^{(1)} = \frac{1}{\sqrt{2}}(\underline{i} - i\underline{j})e^{i\phi} \quad -(15)$$

The e/m potential is:

$$\underline{A}_L^{(1)} = A^{(0)}\underline{q}_L^{(1)} \quad -(16)$$
$$= \frac{A^{(0)}}{\sqrt{2}}(\underline{i} - i\underline{j})e^{i\phi}$$

<u>Through the ECE Ansatz.</u>

<u>Right Circular Polarization</u>

$$V_R^{(1)} = \frac{1}{\sqrt{2}}\begin{bmatrix} 0 \\ 1 \\ i \\ 0 \end{bmatrix}, \quad V^\mu = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix} e^{-i\phi} \quad -(17)$$

Therefore:

$$q_{R\mu}^{(1)} = \frac{1}{\sqrt{2}}\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & i & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} e^{i\phi} \quad -(18)$$

and:

$$\underline{q}_R^{(1)} = \frac{1}{\sqrt{2}}(\underline{i} + i\underline{j})e^{i\phi} \quad -(19)$$

$$\underline{A}_R^{(1)} = \frac{A^{(0)}}{\sqrt{2}}(\underline{i} + i\underline{j})e^{i\phi}.$$

The complex conjugates are obtained straightforwardly from these equations.

4) It is seen that:

$$q^{(1)}_0 = q^{(1)}_z = 0. \quad -(20)$$

Longitudinal Polarization

$$V^{(3)} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \quad V^\mu = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \quad -(21)$$

and:

$$q^{(3)}_\mu = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad -(22)$$

i.e.

$$q^{(3)}_z = 1 \quad -(23)$$

$$\underline{A}^{(3)} = A^{(0)}\underline{k}. \quad -(24)$$

Timelike Polarization

$$V^{(0)} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad V^\mu = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad -(25)$$

and:

$$q^{(0)}_\mu = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad -(26)$$

i.e.

$$q^{(0)}_0 = 1, \quad A^{(0)}_0 = A^{(0)} q^{(0)}_0 \quad -(27)$$

5) It can be seen that $\underline{A}_R^{(1)}$, $A_L^{(1)}$, $\underline{A}^{(3)}$ and $A^{(0)}$ are solutions of the ECE wave equation:

$$\left(\Box + kT \right) A^a_\mu = 0 \quad -(28)$$

in the approximation:

$$kT = \left(\frac{mc}{\hbar} \right)^2 \rightarrow 0. \quad -(29)$$

Here m is the photon mass. It is known experimentally that m is very tiny ($< 10^{-50}$ kgm). So to an excellent approximation:

$$\Box A^a_\mu = 0 \quad -(30)$$

which is the d'Alembert equation. For finite photon mass free of gravitation:

$$\left(\Box + \left(\frac{mc}{\hbar} \right)^2 \right) A^a_\mu = 0 \quad -(31)$$

which is the Proca equation. In ECE theory the Proca equation is generally covariant as required, so is a valid equation of general relativity. This is because the tetrad is generally covariant. There is a problem in the standard model because the Proca equation is not gauge invariant.

5) The origin of circular polarization are the chirality vectors $V^{(1)}_L$ and $V^{(1)}_R$. These are four vectors, representing one time-like and three space-like polarizations. The electromagnetic potential field is within $A^{(0)}$ the tetrad that superimposes the chirality vectors on the spin vectors, V^{μ}.

For the fermion we have seen in notes 70(1) that the tetrad superimposes the chirality vector on the spin vector and gives the Dirac spinor.

For the fermion:

$$a = L, R \quad -(31)$$

$$\mu = 1, 2, \quad -(32)$$

but the overall ~~method~~ is the same, showing that ECE is a unified field ~~theory~~. For electromagnetism the tetrad is 4×4, and for the fermion it is 2×2.

As shown in paper 13 of volume 2, the right and left circular polarization of the electromagnetic field can be described in the same way as the fermion field by a slightly different definition as follows.

6):

$$V^a = \frac{1}{\sqrt{2}} \begin{bmatrix} 1-i \\ 1+i \end{bmatrix}, \quad V^\mu = \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{-i\phi} \quad -(33)$$

this giving the space part of the e/m tetrad as:

$$q_\mu^a = \frac{e^{i\phi}}{\sqrt{2}} \begin{bmatrix} 1 & -i \\ 1 & i \end{bmatrix} \quad -(34)$$

The rows of this tetrad are transposed to columns to give:

$$q_\mu^{aT} = \frac{e^{i\phi}}{\sqrt{2}} \begin{bmatrix} q_X^{(1)} \\ q_Y^{(1)} \\ q_X^{(2)} \\ q_Y^{(2)} \end{bmatrix} \quad -(35)$$

which is a column vector analogous to the Dirac spinor. The electromagnetic potential is:

$$A_\mu^{aT} = A^{(0)} q_\mu^{aT} \quad -(36)$$

and:

$$\Box A_\mu^{aT} = 0 . \quad -(37)$$

This shows that the electromagnetic field can be put into an SU(2) rep., as first shown in the Works by Majorana. This unifies the e/m and fermion fields with ECE.