'68(4) : Resonance in the Newtonian Force

The Newtonian force is defined by :

$$f^a = -m_1 m_2 G \left(R^a{}_b \wedge q^b - \omega^a{}_b \wedge T^b \right) \quad -(1)$$

and the Hodge dual by :

$$\tilde{f}^a = -m_1 m_2 G \left(\tilde{R}^a{}_b \wedge q^b - \omega^a{}_b \wedge \tilde{T}^b \right) \quad -(2)$$

Force due to the Source

In the absence of interaction the force due to the source is :

$$\tilde{f} = -m_1 m_2 G \left(R^0{}_1{}^{10} + R^0{}_2{}^{20} + R^0{}_3{}^{30} \right) \quad -(3)$$

and this is the Newtonian force between two charge carrying masses m_1 and m_2.

Newtonian Force due to the Electric Field.

This is given by :

$$\tilde{f}^a = -m_1 m_2 G \, \omega^a{}_{ib} T^{bi0} \quad -(4)$$

and simplifies to :

2)

$$\tilde{f} = -m_1 m_2 G \underline{\omega}_{int} \cdot \underline{T} \quad -(5)$$

where:

$$\underline{E} = \phi \underline{T} \quad (\text{volts } m^{-1}) \quad -(6)$$

$$= \begin{cases} (\underline{\nabla} \pm \underline{\omega}) \phi \\ \underline{\nabla} \phi \end{cases} \quad -(7)$$

Thus:

$$\left.\begin{aligned} (\underline{\nabla} \pm \underline{\omega}) \phi &= \underline{T} \phi \\ \underline{\nabla} \phi &= \underline{T} \phi \end{aligned}\right\} \quad -(8)$$

Thus:

$$\tilde{f} = \begin{cases} -m_1 m_2 G \underline{\omega}_{int} \cdot \frac{1}{\phi} (\underline{\nabla} \pm \underline{\omega}) \phi \\ -m_1 m_2 G \underline{\omega}_{int} \cdot \frac{1}{\phi} \underline{\nabla} \phi \end{cases} \quad -(9)$$

where:

$$\frac{d^2 \phi}{dr^2} + \left(\frac{1}{r} - \omega_{int} \right) \frac{d\phi}{dr} - \left(\frac{1}{r^2} + \frac{\omega_{int}}{r} \right) \phi = -\frac{\rho}{\epsilon_0}$$

$$-(10)$$

3)

Therefore to maximize the Newtonian force $\tilde{f}$ at resonance: the following term must be maximized

$$\tilde{f} = -m_1 m_2 G \omega_{int} \frac{\phi'}{\phi} \quad -(11)$$

where

$$\phi' = \frac{d\phi}{dr} . \quad -(12)$$

and:

$$\frac{d\phi'}{dr} + \left(\frac{1}{r} - \omega_{int}\right)\phi' - \left(\frac{1}{r^2} + \frac{\omega_{int}}{r}\right)\phi = -\frac{\rho}{\epsilon_0} . \quad -(13)$$

Therefore, for a given ϕ, ϕ' must be maximized through the resonance equation:

$$\boxed{\frac{d^2\phi'}{dr} + \frac{d}{dr}\left(\left(\frac{1}{r} - \omega_{int}\right)\phi'\right) - \frac{d}{dr}\left(\left(\frac{1}{r^2} + \frac{\omega_{int}}{r}\right)\phi\right) = -\frac{d\rho}{dr} \Big/ \epsilon_0 \quad -(14)}$$

This equation is:

4)

$$\frac{d^2\phi'}{dr^2} + \left(\frac{1}{r} - \omega_{int}\right)\frac{d\phi'}{dr} - \left(\frac{1}{r^2} + \frac{\omega_{int}}{r}\right)\frac{d\phi'}{dr}$$

$$+ \left(\frac{d}{dr}\left(\frac{1}{r} - \omega_{int}\right)\right)\phi' - \left(\frac{d}{dr}\left(\frac{1}{r^2} + \frac{\omega_{int}}{r}\right)\right)\phi$$

$$= -\frac{d\rho}{dr} \Big/ \epsilon_0 \quad -(15)$$

i.e. :

$$\frac{d^2\phi'}{dr^2} + \left(\frac{1}{r} - \omega_{int} - \frac{1}{r^2} - \frac{\omega_{int}}{r}\right)\frac{d\phi'}{dr}$$

$$- \left(\frac{1}{r^2} + \frac{d\omega_{int}}{dr}\right)\phi' + \left(\frac{2}{r^3} + \frac{\omega_{int}}{r^2} - \frac{1}{r}\frac{d\omega_{int}}{dr}\right)\phi$$

$$= -\frac{d\rho}{dr} \Big/ \epsilon_0 \quad -(16)$$

For a given ϕ, ϕ' is maximized at resonance from eq. (16), and so $\bar{f}$ is maximized by eq. (11).