67(5): Complex Circular Basis for the Spin Connection

The complex circular basis is a convenient representation of the internal indices a, b and c because of the angular momentum property described in note 67(4), i.e.:

$$\hat{J}^{(3)} \underline{e}^{(1)} = -1\, \underline{e}^{(1)} \quad -(1)$$

$$\hat{J}^{(3)} \underline{e}^{(2)} = 1\, \underline{e}^{(2)} \quad -(2)$$

$$\hat{J}^{(3)} \underline{e}^{(3)} = 0\, \underline{e}^{(3)} \quad -(3)$$

where:

$$\hat{J}^{(3)} = i\, \underline{e}^{(3)} \times . \quad -(4)$$

The boson electric field can be defined by:

$$\underline{E}^{(1)} = -\underline{\nabla}\phi - \underline{\omega}^{(1)}\phi \quad -(5)$$

$$\underline{E}^{(2)} = -\underline{\nabla}\phi + \underline{\omega}^{(2)}\phi \quad -(6)$$

$$\underline{E}^{(3)} = -\underline{\nabla}\phi . \quad -(7)$$

Here the spin connection is defined by:

$$\underline{\omega}^{(1)} = \frac{1}{r}\underline{e}^{(1)} \quad -(8)$$

$$\underline{\omega}^{(2)} = \frac{1}{r}\underline{e}^{(2)} \quad -(9)$$

$$\underline{\omega}^{(3)} = 0\, \underline{e}^{(3)} . \quad -(10)$$

The boson structure of the electric field is

2) then demonstrated by:

$$\boxed{\begin{aligned} \hat{J}^{(3)} \underline{E}^{(1)} &= -\phi \, \underline{\omega}^{(1)} \\ \hat{J}^{(3)} \underline{E}^{(2)} &= +\phi \, \underline{\omega}^{(2)} \\ \hat{J}^{(3)} \underline{E}^{(3)} &= \phi \, \underline{0} \end{aligned}} \quad -(11)$$

Static Limit of the Circularly Polarized Electric Field.

The circularly polarized electric field is:

$$\underline{E}^{(1)} = \frac{E^{(0)}}{\sqrt{2}} (\underline{i} - i\underline{j}) \exp(i(\omega t - \kappa Z)) \quad -(12)$$

$$\underline{E}^{(2)} = \frac{E^{(0)}}{\sqrt{2}} (\underline{i} + i\underline{j}) \exp(-i(\omega t - \kappa Z)) \quad -(13)$$

The static limit can be defined as:

$$\phi \rightarrow 0 \quad -(14)$$

where:

$$\phi = \omega t - \kappa Z . \quad -(15)$$

In this case:

$$\left.\begin{aligned} \mathrm{Real}(\underline{E}^{(1)}) = \mathrm{Real}(\underline{E}^{(2)}) &= \frac{E^{(0)}}{\sqrt{2}} \underline{i} \\ \mathrm{Im}(\underline{E}^{(1)}) = -\mathrm{Im}(\underline{E}^{(2)}) &= -\frac{E^{(0)}}{\sqrt{2}} \underline{j} . \end{aligned}\right\} -(16)$$

Therefore the spin connection for plane

3) waves is:

$$\underline{\omega}^{(1)} = \frac{\omega^{(0)}}{\sqrt{2}}\left(\underline{i} - i\underline{j}\right)\exp\left(i\left(\omega t - \kappa z\right)\right) \quad -(17)$$

$$\underline{\omega}^{(2)} = \frac{\omega^{(0)}}{\sqrt{2}}\left(\underline{i} + i\underline{j}\right)\exp\left(-i\left(\omega t - \kappa z\right)\right) \quad -(18)$$

This means that the frame itself is both spinning and translating.