

65(3): A Simple Example of Resonance in Magneto-statics

Start with:

$$\nabla \times (\nabla \times \underline{A}^a - g \underline{A}^b \times \underline{A}^c) = \mu_0 \underline{J}_m^a \quad - (1)$$

and we:

$$\nabla \times (\nabla \times \underline{A}^a) = -\nabla^2 \underline{A}^a + \nabla (\nabla \cdot \underline{A}^a) \quad - (2)$$

$$\nabla \times (\underline{A}^b \times \underline{A}^c) = \underline{A}^b \nabla \cdot \underline{A}^c - \underline{A}^c \nabla \cdot \underline{A}^b + (\underline{A}^c \cdot \nabla) \underline{A}^b - (\underline{A}^b \cdot \nabla) \underline{A}^c \quad - (3)$$

Assume that:

$$\nabla \cdot \underline{A}^a = \nabla \cdot \underline{A}^b = \nabla \cdot \underline{A}^c = 0, \quad - (4)$$

$$(\underline{A}^b \cdot \nabla) \underline{A}^c = \underline{0}. \quad - (5)$$

Then:

$$\nabla^2 \underline{A}^a + (\underline{A}^c \cdot \nabla) \underline{A}^b = -\mu_0 \underline{J}_m^a \quad - (6)$$

Considering the z component:

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) A_z^a + \left(A_x^c \frac{\partial}{\partial x} + A_y^c \frac{\partial}{\partial y} + A_z^c \frac{\partial}{\partial z} \right) A_z^b = -\mu_0 J_{mz}^a \quad - (7)$$

$$\text{If: } A_z^c = A_z^b \quad - (8)$$

2) and $\frac{\partial A_z^a}{\partial z} = \frac{\partial A_z^a}{\partial t} = 0$ — (a)

Der: $\frac{\partial^2 A_z^a}{\partial x^2} + \left(\frac{\partial A_z^b}{\partial x} \right) A_z^a = -\mu_0 J_{mz}^a$ — (10)

If $\frac{\partial A_z^b}{\partial x} = \kappa_0^2$ — (11)

eq (10) becomes:

$$\frac{\partial^2 A_z^a}{\partial x^2} + \kappa_0^2 A_z^a = \mu_0 J_{mz}^a(0) \cos(\kappa_0 x)$$
 — (12)

if: $J_{mz}^a = -J_{mz}^a(0) \cos(\kappa_0 x)$ — (13)

The solution of eq. (12) is:

$$A_z^a = \frac{\mu_0 J_{mz}^a(0) \cos(\kappa x)}{\kappa_0^2 - \kappa^2}$$
 — (14)

Resonance occurs at: $\kappa_0 = \kappa$ — (15)

Eq. (8) is compatible with

$$\underline{\nabla} \cdot \underline{A}^b \times \underline{A}^a = 0,$$
 — (16)

$$\underline{A}^b = \underline{A}^a.$$
 — (17)