64(10): Overall Method for Magnetism (Magnetostatics)

Following the method developed in the Coulomb Law analysis of paper 63, the ECE laws of magnetism must be developed in general with simultaneous equations. For magnetostatics these are:

$$\underline{\nabla} \cdot \underline{B}^a = \mu_0 j^{0a} \quad -(1)$$

$$\underline{\nabla} \times \underline{B}^a = \mu_0 \underline{J}_m^a \quad -(2)$$

where:

$$\underline{B}^a = \underline{\nabla} \times \underline{A}^a - \underline{\omega}^a{}_b \times \underline{A}^b. \quad -(3)$$

The equivalent standard model laws are ($\underline{B} = \underline{\nabla} \times \underline{A}$):

$$\underline{\nabla} \cdot \underline{B} = 0 \quad -(4)$$

$$\underline{\nabla} \times \underline{B} = \mu_0 \underline{J}. \quad -(5)$$

In developing the ECE Coulomb Law the spin connection was found in the off resonant condition and shown to be also a valid solution under all conditions. SCR was then demonstrated for electrostatics. The same method must now be used for magnetostatics. The structure of $\underline{\omega}^a{}_b$ is found as in notes 64(8).

When rotation and translation are not mutually influential:

$$j^{0a} = 0. \quad -(6)$$

In the off-resonance condition:

2)

$$\underline{\nabla} \times \underline{A}^a = -\underline{\omega}^a{}_b \times \underline{A}^b \quad -(7)$$

From eq. (6) in eq. (1):

$$\boxed{\underline{\nabla} \cdot (\underline{\omega}^a{}_b \times \underline{A}^b) = 0.} \quad -(8)$$

Eqs. (7) and (8) are simultaneous equations, but are not enough to determine $\underline{\omega}^a{}_b$ and $\underline{A}^b$ because eq. (7) in eq. (8) gives an identity:

$$\underline{\nabla} \cdot (\underline{\nabla} \times \underline{A}^a) = 0 \quad -(9)$$

Eq (9) is always true for any $\underline{\omega}^a{}_b$ and $\underline{A}^b$.

However, eq. (8) and:

$$\boxed{\underline{\nabla} \times (\underline{\nabla} \times \underline{A}^a - \underline{\omega}^a{}_b \times \underline{A}^b) = \mu_0 \underline{J}_m^a} \quad -(10)$$

are true simultaneous equations. Provided $\underline{J}_m^a$ is known or modelled, $\underline{A}^a$ and $\underline{\omega}^a{}_b$ can be found separately from eqns. (8) and (10).

A possible solution of eq. (8) is:

$$\underline{\omega}^a{}_b \times \underline{A}^b = \underline{0}, \quad -(11)$$

but this leads back to the standard model, and no SCR in magnetostatics.

Therefore the method adopted is as follows.

3) Eq. (7) is used in the off-resonance condition to find a particular solution for $\underline{\omega}^a{}_b$ given a form for $\underline{A}^a$ in the off-resonant condition. This solution is also a solution of eq. (8) automatically, because eq. (9) is always true. Finally use ~~this~~ particular spin connection in the resonance equation (10), in which $\underline{A}^a$ is such that eq (7) is not true in general.

This parallels the treatment of the spin connection in paper 63 for the Coulomb Law. Due to the assumption (6), the spin connection vector is:

$$\underline{\omega} = \omega^2{}_3 \, \underline{i} + \omega^1{}_3 \, \underline{j} + \omega^1{}_2 \, \underline{k} \quad -(12)$$

where:

$$\omega^1{}_2 = \frac{\kappa}{2}\left(q^0 + q^3\right) \quad -(13)$$

$$\omega^1{}_3 = \frac{\kappa}{2}\left(-q^2 + q^0\right) \quad -(14)$$

$$\omega^2{}_3 = \frac{\kappa}{2}\left(q^1 + q^0\right) \quad -(15)$$

and:

$$A^a = A^{(0)} q^a. \quad -(16)$$

<u>This states the problem to be solved.</u>