1)

63(2): The Complete Resonance Solutions

These are:

$$\frac{d^2\phi}{dZ^2} + \frac{1}{Z_0^2}\phi = \frac{\rho_0}{\epsilon_0}\cos(\kappa' x) \quad -(1)$$

where:

$$\cos(\kappa' x) = -\cos(2\kappa_0 x)\left(\cos\left(\kappa Z_0\left(\cos\left(\kappa_0 x + \cosh\left(\sin\left(\kappa_0 x\right)\right)\right)\right)\right)\right) \quad -(2)$$

and from:

$$2\kappa_0 \frac{d\phi}{dx} = \frac{\rho_0}{\epsilon_0}\,\mathrm{Im}\left(e^{2i\kappa_0 x}\cos\left(\kappa Z_0 e^{i\kappa_0 x}\right)\right) \quad -(3)$$

we obtain:

$$\frac{d\phi}{dx} = \frac{\rho}{2\epsilon_0} Z_0 \sin(2\kappa_0 x)\left(\cos\left(\kappa Z_0\left(\cos\left(\kappa_0 x + \cosh\left(\sin\left(\kappa_0 x\right)\right)\right)\right)\right)\right) \quad -(4)$$

Amplitude and kinetic energy resonance occurs from eq. (1) at:

$$\kappa_R = \kappa_E = 1/Z_0 \quad -(5)$$

The analytical solution of eq. (1) is:

$$\phi = \frac{\rho_0}{\epsilon_0}\frac{\cos(\kappa' Z)}{\left(\frac{1}{Z_0^2} - \kappa'^2\right)} \quad -(6)$$

)

i.e :

$$\phi = -\frac{\rho_0}{\epsilon_0} \frac{\left(\cos(2k_0 z)\right)\left(\cos\left(k z_0 \left(\cos\left(k_0 z + \cosh\left(\sin(k_0 z)\right)\right)\right)\right)\right)}{\left(\frac{1}{z_0^2} - k'^2\right)} \qquad -(7)$$

where :

$$k' = -\frac{1}{z} \cos^{-1}\left(\left(\cos(2k_0 z)\right)\left(\cos\left(k z_0 \left(\cos\left(k_0 z + \cosh\left(\sin(k_0 z)\right)\right)\right)\right)\right)\right) \qquad -(8)$$

At resonance :

$$k'^2 = \frac{1}{z_0^2} \qquad -(9)$$

and

$$\phi \rightarrow \infty \qquad -(10)$$