15) : Optimal Resonance Equation.

Start with:

$$\frac{d^2\phi}{dz^2} - \frac{1}{z}\frac{d\phi}{dz} + \frac{\phi}{z^2} = -\frac{\rho(0)}{\epsilon_0}\cos(\kappa z) \quad -(1)$$

This can be solved numerically as it is if preferred. Here is only one parameter, κ, apart from the scaling factor $\rho(0)/\epsilon_0$. If preferred, the Euler method can be used to transform eq (1) into a resonance equation with constant coefficients. The transformation of variables in the Euler method is:

$$z = z_0 e^{\kappa_1 x} \quad -(2)$$

in general. This introduces two new parameters, z_0 and κ_1. Following a textbook such as Stephenson, eq. (2) in eq (1) produces:

$$\frac{d^2\phi}{dx^2} - 2\kappa_1\frac{d\phi}{dx} + \kappa_1^2\phi = -\frac{\rho(0)}{\epsilon_0}\kappa_1^2\cos\left(\kappa z_0 e^{\kappa_1 x}\right) \quad -(3)$$

In eq. (3) there are three parameters, κ, κ_1 and z_0. If preferred these three parameters may be used as independent parameters in the code. However, one may simplify to one parameter, κ_1 using:

$$\kappa z_0 = 1. \quad -(4)$$