Notes 63(10)

Simple solution of the Resonant Hartree Equation

If we start with eq (25) of notes 63(9) with a positive spin connection sign:

$$(x^2+y^2+z^2)\nabla^2\phi - x\frac{\partial\phi}{\partial x} - y\frac{\partial\phi}{\partial y} - z\frac{\partial\phi}{\partial z} + \phi$$
$$= -(x^2+y^2+z^2)\frac{\rho_0}{\epsilon_0}\cos(\underline{\kappa}\cdot\underline{r}) \quad -(1)$$

and use:

$$\left.\begin{aligned} x &= x_0 e^{i\underline{\kappa}\cdot\underline{r}} \\ y &= y_0 e^{i\underline{\kappa}\cdot\underline{r}} \\ z &= z_0 e^{i\underline{\kappa}\cdot\underline{r}} \end{aligned}\right\} \quad -(2)$$

Then $\underline{R}\cdot\underline{R}^* = x_0^2+y_0^2+z_0^2 = \frac{1}{\kappa_0^2}$ $-(3)$

and eq (1) becomes:

$$\nabla^2\phi + \kappa_0^2\phi = -\frac{\rho_0}{\epsilon_0}\cos(\underline{\kappa}\cdot\underline{r}) \quad -(4)$$

which is a simple undamped oscillator.

For each particle: $-(5)$

$$\nabla^2\phi + \kappa_{0,i}^2\phi = -\frac{\rho_0}{\epsilon_0}\cos(\underline{\kappa}_i\cdot(\underline{r}-\underline{r}_i))$$

$$\rho(\underline{r}) = \sum_{i=1}^{n} e_i\,\delta(\underline{r}-\underline{r}_i)$$