62(3): Further Testing of the ECE Lemma

The ECE Lemma is derived from the tetrad postulate:

$$D_\mu q^a_\nu = 0 \qquad -(1)$$

through the identity:

$$D^\mu (D_\mu q^a_\nu) = \partial^\mu (D_\mu q^a_\nu) := 0. \qquad -(2)$$

The tetrad postulate is

$$D_\mu q^a_\lambda = \partial_\mu q^a_\lambda + \omega^a_{\mu b} q^b_\lambda - \Gamma^\nu_{\mu\lambda} q^a_\nu = 0 \qquad -(3)$$

Eq (3) follows from the rule for covariant differentiation of a rank two mixed index tensor, q^a_λ.

From eqs. (2) and (3):

$$\partial^\mu \left(\partial_\mu q^a_\lambda + \omega^a_{\mu b} q^b_\lambda - \Gamma^\nu_{\mu\lambda} q^a_\nu \right) = 0 \qquad -(4)$$

i.e. $$\Box q^a_\lambda = \partial^\mu \left(\Gamma^\nu_{\mu\lambda} q^a_\nu - \omega^a_{\mu b} q^b_\lambda \right) \qquad -(5)$$

Now define:

$$R q^a_\lambda := \partial^\mu \left(\Gamma^\nu_{\mu\lambda} q^a_\nu - \omega^a_{\mu b} q^b_\lambda \right) \qquad -(6)$$

and use $$q^a_\lambda q^\lambda_a = 1 \qquad -(7)$$

2) to find:

$$R = q^{\lambda}_{a}\, \partial^{\mu}\left(\Gamma^{\nu}_{\mu\lambda}\, q^{a}_{\nu} - \omega^{a}_{\mu b}\, q^{b}_{\lambda}\right) \quad -(8)$$

and

$$\boxed{\Box q^{a}_{\lambda} = R\, q^{a}_{\lambda}} \quad -(9)$$

We may further define:

$$\Gamma^{a}_{\mu\nu} = \Gamma^{\nu}_{\mu\lambda}\, q^{a}_{\nu}\, , \quad \omega^{a}_{\mu\lambda} = \omega^{a}_{\mu b}\, q^{b}_{\lambda} \quad -(10)$$

and

$$R = q^{\nu}_{a}\, R^{a}_{\nu} \quad -(11)$$

where:

$$R^{a}_{\nu} = \partial^{\mu}\left(\Gamma^{a}_{\mu\nu} - \omega^{a}_{\mu\nu}\right). \quad -(12)$$

<u>Limit of Zero R</u>

If $R = 0$, it follows that

$$\partial^{\mu}\left(\Gamma^{a}_{\mu\nu} - \omega^{a}_{\mu\nu}\right) = 0 \quad -(13)$$

so

$$\Gamma^{a}_{\mu\nu} = \omega^{a}_{\mu\nu} + \zeta^{a}_{\mu\nu}(0) \quad -(14)$$

where $\zeta^{a}_{\mu\nu}(0)$ is a constant independent of x^{μ}. It follows that in field theory:

$$\zeta^{a}_{\mu\nu}(0) = 0 \quad -(15)$$

3) because if $\gamma^a_{\mu\nu}(0)$ is independent of x^μ it cannot represent curvature or torsion. Any field must be curvature or torsion or a combination thereof.

So if:

$$R = 0 \quad -(16)$$

then:

$$\Gamma^a_{\mu\nu} = \omega^a_{\mu\nu} \quad -(17)$$

and from eq. (8):

$$\partial_\nu q^a_\mu = 0. \quad -(18)$$

In gravitational theory this means that q^a_μ becomes independent of x^ν, indicating no curvature (i.e. $R = 0$ self-consistently). This means no gravitation.

In electromagnetic theory eq. (18) means:

$$\partial_\nu A^a_\mu = 0 \quad -(19)$$

or

$$\partial^\nu A^a_\mu = 0. \quad -(20)$$

Thus A^a_μ is independent of x^ν, meaning a constant electromagnetic potential and no torsion, no electric field and no magnetic field.

This in turn means that ECE theory is

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electrodynamics means that there must be a finite R. In the limit of special relativity the gravitational field has no influence on the e/m field and:

$$R = -\left(\frac{mc}{\hbar}\right)^2 \quad -(21)$$

where m is the photon mass. This is Einstein's equivalence principle. In this limit the ECE Lemma reduces to the Proca equation:

$$\left(\Box + \frac{m^2 c^2}{\hbar^2}\right) A_\mu^a = 0 \quad -(22)$$

where m is identically non-zero for finite electric and magnetic fields.

If gravitation and e/m interact as in NASA Cassini, then eqn. (22) becomes:

$$\left(\Box + kT\right) A_\mu^a = 0 \quad -(23)$$

where both the e/m and gravitational fields contribute to T, the index reduced canonical energy-momentum density.