

SPACE-TIME RESONANCES IN THE COULOMB LAW

by

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ABSTRACT

The Coulomb law is derived from general relativity applied to classical electrodynamics within Einstein Cartan Evans (ECE) unified field theory. The radial component of the spin connection is modeled to be of the form $1/r$, where r is the radial component of the spherical polar coordinate system. The Coulomb potential so obtained may be amplified by space-time resonance. If this resonant Coulomb potential is used in a computation of the radial orbitals of the H atom, for example, the latter ionizes if the kinetic energy inputted from space-time at resonance exceeds the ionization potential energy (13.6 eV). The free electrons so released may be used as a novel source of electric power.

Keywords: Einstein Cartan Evans (ECE) field theory, resonant Coulomb law, radial orbitals of the H atom, free electrons from resonance, source of electric power from space-time.

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1. INTRODUCTION

The theory of general relativity was developed for the gravitational field, as is well known, and has recently been tested in the solar system {1} to one part in one hundred thousand with the NASA Cassini experiments. It is therefore logical to extend general relativity to other areas of physics, notably classical electrodynamics, thereby developing a generally covariant unified field theory {2-18} for the natural, engineering and life sciences. In the standard model, classical electrodynamics is a theory of special relativity - the Maxwell Heaviside (MH) field theory {19}. The well known Coulomb law is part of the MH field theory and is usually regarded as one of the most precise laws in physics {19, 20}. The Coulomb law is the basis for the quantum theory of atomic and molecular spectra for example, and is used in many of the advanced computational techniques employed in this area of physics and chemistry. When the MH theory is extended from special to general relativity {2-18} with the Einstein Cartan Evans (ECE) theory, important new features develop in all the basic laws of classical electrodynamics, including the Coulomb law. These features emanate from the spin connection of ECE space-time. The Minkowski space-time of the MH theory is the well known flat space-time {19} of special relativity, but ECE space-time is characterized by the presence of both curvature and torsion {2-18}. In general relativity (ECE theory) the electromagnetic field is spinning space-time and the gravitational field is curving space-time. The spinning and curving may interact through standard Cartan geometry {21} and therefore the electromagnetic and gravitational fields may interact as verified experimentally in the well known bending of light by gravity. This phenomenon has been observed with great precision in the recent NASA Cassini experiments. ECE theory has been accepted {22} as the first classical explanation of this phenomenon {2-18}. The original well known inference of this effect by Einstein and others is based on a semi-classical approach, where the photon mass gravitates with the mass of the sun according to the Einstein

Hilbert (EH) field theory of gravitation published in 1916. A classical explanation was not possible prior to ECE theory because standard model electrodynamics is special relativity un-unified with gravitational general relativity. ECE theory {2-18} provides a relatively simple and practical unified field theory based on the fundamental and well known principle of general covariance {21}. Unification occurs on both classical and quantum levels, and so ECE theory has been accepted as unifying general relativity with quantum mechanics, a major aim of physics throughout the twentieth century.

In Section 2 the Coulomb law is developed within the context of ECE field theory using a simple model of the spin connection, which is assumed to have a $1 / r$ radial dependence, where (r, θ, ϕ) is the spherical polar coordinate system {23}. The result is that the Poisson equation is extended to a second order differential equation through which the scalar potential may be amplified at resonance according to well known mathematical principles {24}. This capacity for resonance is due to the presence of the spin connection of ECE space-time itself. Resonance of this type is not possible in a flat space-time, because in a flat space-time there is no spin connection. The latter indicates that the electromagnetic field is spinning space-time. The latter inference is indicated independently by several other phenomena {2-18}, notably the magnetization of matter by electromagnetic radiation (the inverse Faraday effect) and the presence of the ECE spin field (B(3) {25}) in all types of electromagnetic radiation. The inverse Faraday effect is magnetization due to the B(3) spin field. The latter originates {2-18} in the spin connection, which works its way into other observable phenomena throughout the whole of the natural, engineering and life sciences.

In Section 3 some graphical results are given from the resonant Coulomb law, and it is shown how this produces free electrons from the H atom by ionizing the latter with kinetic energy inputted from space-time at resonance. The H atom is used here as a simple model material. The release of free electrons at space-time resonance has been observed

recently {26} and shown to be a repeatable phenomenon. The material and circuit designs used in this series of experiments {26} are much more complicated than H, but the latter serves as a model to illustrate the theoretical principles at work - those of general relativity applied to classical electrodynamics with ECE theory.

2. THE ECE RESONANCE COULOMB LAW

The law is given {2-18} from the first Cartan structure equation:

$$T^a = d \wedge q^a + \omega^a_b \wedge q^b \quad - (1)$$

and the first Bianchi identity:

$$d \wedge T^a + \omega^a_b \wedge T^b = R^a_b \wedge q^b \quad - (2)$$

with the ECE Ansatz:

$$A^a = A^{(0)} q^a, \quad F^a = A^{(0)} T^a \quad - (3)$$

Here T^a is the torsion form, R^a_b is the Riemann or curvature form, q^a is the tetrad form, ω^a_b is the spin connection form, $A^{(0)}$ is the electromagnetic potential form, $cA^{(0)}$ is the primordial voltage, and F^a is the electromagnetic field form. The Ansatz was first proposed by Cartan in well known correspondence with Einstein in the first part of the twentieth century, but was not developed into ECE theory until the spring of 2003 {2-18}. Eqs. (1) to (3) lead to {2-18}:

$$\underline{E}^a = - \partial A^a / \partial t - \underline{\nabla} \phi^a - c \omega^{0a}_b A^b + \phi^b \omega^a_b, \quad - (4)$$

$$\underline{\nabla} \cdot \underline{E}^a = c \mu_0 \tilde{J}^{0a}, \quad - (5)$$

in vector notation. Here \underline{E}^a is the electric field strength (volts per meter), μ_0 is the vacuum

S.I. permeability, \tilde{J}^{0a} is the time-like component of the inhomogeneous four-current of ECE theory, c is the vacuum speed of light, \underline{A}^a is the vector potential, ϕ^b is the scalar potential, ω^{0a} is the time-like part of the spin connection four-vector, and $\underline{\omega}^a_b$ is the space-like part of the spin connection four-vector. The indices a and b originate in Cartan geometry {2-18, 21} and are the indices of the tangent space-time at a point P in the base manifold. These indices indicate polarization states of electromagnetic radiation in ECE theory {2-18}. Eq. (5) may be written for each index a as:

$$\underline{\nabla} \cdot \underline{E} = \rho / \epsilon_0 \quad - (6)$$

where ρ is the charge density and where ϵ_0 is the S.I. vacuum permittivity. Therefore for each index a , Eq. (5) has the same mathematical structure as the standard model Coulomb law {19, 20}. However, the electric field in ECE theory must always be defined by Eq (4), which always involves the spin connection. The electric field is part of spinning space-time.

If attention is restricted to the scalar potential, then for each index a , Eq. (4) is:

$$\underline{E} = -\underline{\nabla} \phi + \phi^b \underline{\omega}_b \quad - (7)$$

Here ϕ^b is interpreted as a scalar quantity indexed or labeled by b , indicating that the scalar potential applies to this state of polarization of electromagnetic radiation. For a given b index, Eq. (7) is:

$$\underline{E} = -\underline{\nabla} \phi + \phi \underline{\omega} \quad - (8)$$

Summation over repeated b indices in Eq. (7) is implied (Einstein convention) but for the sake of simplicity it has been assumed in Eq. (8) that there is only one index and one state of polarization. Therefore we have reduced the complicated Eq. (4) to its simplest form

(8). The result is that the familiar definition of the electric field in the standard model

Coulomb law:

$$\underline{E} = -\underline{\nabla} \phi \quad - (9)$$

is supplemented by a term in the vector part of the spin connection, the vector $\underline{\omega}$. Eqs.

(6) and (8) give the second order differential equation:

$$\nabla^2 \phi - \underline{\nabla} \cdot (\phi \underline{\omega}) = -\rho / \epsilon_0 \quad - (10)$$

which compares with the standard model Poisson equation {19, 20}:

$$\nabla^2 \phi = -\rho / \epsilon_0 \quad - (11)$$

Eq. (10) is an equation of general relativity. Eq. (11) is an equation of special relativity.

The mathematical properties of Eq. (10) include the ability to give resonance, whereas Eq.

(11) has no resonance solutions. This is a key difference. Resonance is the key to the

production of free electrons from ECE space-time, providing a new source of electric power

for engineering.

The spin connection vector in Cartesian and spherical polar coordinates is:

$$\begin{aligned} \underline{\omega} &= \omega_x \underline{i} + \omega_y \underline{j} + \omega_z \underline{k} \\ &= \omega_r \underline{e}_r + \omega_\phi \underline{e}_\phi + \omega_\theta \underline{e}_\theta \end{aligned} \quad - (12)$$

where ω_r is the radial component of $\underline{\omega}$. If the latter is assumed to be purely radial, for simplicity of argument, then:

$$\underline{\omega} = \omega_r \underline{e}_r \quad - (13)$$

and in spherical polar coordinates {23}:

$$\underline{\omega} \cdot \underline{\nabla} \phi = \omega_r \frac{\partial \phi}{\partial r}, \quad - (14)$$

$$\phi \underline{\nabla} \cdot \underline{\omega} = \frac{\phi}{r^2} \frac{\partial}{\partial r} (r^2 \omega_r), \quad - (15)$$

$$\nabla^2 \phi = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial \phi}{\partial r} \right) = \frac{\partial^2 \phi}{\partial r^2} + \frac{2}{r} \frac{\partial \phi}{\partial r}. \quad - (16)$$

The dimensions of $\underline{\omega}$ are inverse meters {2-18}, so the simplest model of the vector spin connection is:

$$\omega_r = \frac{A}{r} \quad - (17)$$

where A is a dimensionless scaling factor. Eqs. (13) to (17) give the result:

$$\frac{\partial^2 \phi}{\partial r^2} + (2-A) \frac{1}{r} \frac{\partial \phi}{\partial r} - \frac{A\phi}{r^2} = \frac{-\rho}{\epsilon_0} \quad - (18)$$

in spherical polar coordinates. Eq. (18) contains second and first order partial derivatives in the scalar potential ϕ . In the special case

$$A = 2 \quad - (19)$$

Eq. (18) becomes:

$$\frac{\partial^2 \phi}{\partial r^2} - \frac{2\phi}{r^2} = \frac{-\rho}{\epsilon_0} \quad - (20)$$

in which the second term on the left hand side is a REPULSION term. This means that the familiar Coulomb attraction between a proton and an electron in an H atom develops a repulsive component due to the presence of the spin connection vector. Eq. (18) has a similar structure to the well known one-dimensional Schrödinger equation for motion in an

effective potential with repulsive centrifugal term {20} in the H atom. So the spin connection may be interpreted similarly. If the repulsion term in Eq. (20) becomes strong enough, the H atom ionizes, releasing a free electron. Eq. (18) is similar to the well-known {24} class of linear inhomogeneous differential equations that give resonance - the damped driven oscillator equations. Eq. (20) is a special case - the undamped driven oscillator. In order to induce resonance, the charge density ρ must be initially oscillatory {24}. In the H atom model we are considering the source of this small original oscillation may be considered to be zitterbewegung (jitterbugging) from quantum electrodynamics {20}. In a molecule it could be a rotational frequency or vibrational bond frequency. At space-time resonance the initially small oscillation is greatly amplified {2-18, 24} and kinetic energy is absorbed into the atom or molecule from ECE space-time. If this energy is greater than the ionization potential energy of H (13.6 eV) the electron breaks free of the proton and may be used in a circuit to produce electric power from space-time through the intermediacy of the H atom. This concept may be generalized to any material which contains electrons which are easily released by ionization. The skill in material design revolves around this need. The engineering skill consists in devising a design to induce the resonance and this has been accomplished recently in a repeatable manner {26}. The output power in such experiments {26} may exceed the input power by as much as a factor of one hundred thousand, an amplification that illustrates dramatically the resonance of the spin connection in classical electrodynamics. Care has been taken to ensure that this experiment is repeatable and the apparatus has been observed independently {26} in different laboratories. Every effort has been made to eliminate artifact, and reproducible amplification by five orders of magnitude is unlikely to be artifact. The standard model (MH theory) has no explanation for this phenomenon, even on a qualitative level. Its explanation in general relativity (ECE theory) relies on resonating the spin connection as described already.

In summary of this section therefore the Poisson equation of the standard model

(Eq. (11)) is modified in the simplest instance to the following ECE equation of general relativity:

$$\frac{\partial^2 \phi}{\partial r^2} = \frac{2\phi}{r^2} - \frac{\rho}{\epsilon_0} \quad (21)$$

introducing a repulsive term:

$$\rho_{\text{eff}} = 2\epsilon_0 \frac{\phi}{r^2} \quad (22)$$

If the charge density ρ is very small, Eq. (21) takes on the approximate mathematical form:

$$\frac{\partial^2 \phi}{\partial r^2} \sim \frac{2\phi}{r^2} \quad (23)$$

which has an analytical solution:

$$\phi \sim \frac{\beta}{r} + \alpha r^2 \quad (24)$$

where α and β are constants. When r is very small, the potential ϕ becomes very large and a large amount of POSITIVE potential energy may be inputted into the H atom from the spin connection, depending on the value of β . If:

$$\beta \gg \frac{e}{4\pi\epsilon_0} \quad (25)$$

then the positive repulsion potential becomes equal to or greater than the negative attraction potential, releasing the electron from the proton. The standard model inverse square Coulomb law is very precise in the vast majority of experiments in macroscopic classical electrodynamics {19} but the recent experiments carried out in ref. (26) indicate that it does not hold in general.

The ECE theory reduces straightforwardly to the standard Coulomb law as follows.

In ECE theory the electric field is defined in the simplest instance by:

$$\underline{E} = -\underline{\nabla} \phi + \phi \underline{\omega} \quad - (26)$$

and in the standard Coulomb law it is defined by:

$$\underline{E} = -\underline{\nabla} \phi. \quad - (27)$$

Therefore if:

$$\underline{\nabla} \phi = -\phi \underline{\omega} \quad - (28)$$

the mathematical form of the standard Coulomb law is obtained:

$$\underline{E} := -2 \underline{\nabla} \phi. \quad - (29)$$

This simply means that the scalar potential is defined by:

$$\underline{\Phi} := 2\phi. \quad - (30)$$

This makes no difference to the observable force (inverse square law). If:

$$\underline{\Phi} := \frac{e}{4\pi \epsilon_0 r^2} \quad - (31)$$

and

$$\underline{\nabla} \underline{\Phi} = -\underline{\Phi} \underline{\omega} \quad - (32)$$

then:

$$\omega_z = \frac{1}{z} \quad - (33)$$

The important conclusion is reached that a spin connection of the type (33) is ALWAYS observed in the Coulomb law, which becomes a law of general relativity as required by objectivity in physics. Therefore any experimental departure from the inverse square Coulomb law would indicate that the spin connection is no longer given by Eq. (33). In general relativity (ECE theory) the electric field must always be defined according to Eq. (26) and the vast majority of experimental data have confirmed the inverse square law of Coulomb for over two hundred years. In general relativity this means that the data show that the spin connection must be of the form (33) experimentally. This type of spin connection, conversely, gives the inverse square law of Coulomb. Eq. (28) is similar to the operator equivalence of quantum mechanics, and means that:

$$\underline{\omega} \rightarrow -\underline{\nabla} \quad - (34)$$

The operator equivalence is:

$$\underline{p} \rightarrow -i\hbar\underline{\nabla} \quad - (35)$$

Therefore in general relativity the electric field can be defined equivalently in two ways:

$$\underline{E} = -\underline{\nabla} \Phi = \Phi \underline{\omega} \quad - (36)$$

and this is the fundamental definition of the electric field in general relativity. These considerations confirm that ECE theory is correct to very high precision, and give a simple meaning to the spin connection. The result of general relativity, eq. (26), is preferred to the result of special relativity, eq. (27), on several grounds, notably that other aspects of electrodynamics such as the inverse Faraday effect and Eddington effect require a generally

covariant unified field theory for their objective interpretation on the classical level.

Dramatically new results such as those by the Mexican group are also accounted for by ECE theory by considerations of resonance as in previous work.

It is significant that spin connections of the type (33) also occur as Christoffel connections of the Schwarzschild metric of spherically symmetric space-time. These are well known to indicate a dynamic space-time. Electrodynamics is also now known to be a phenomenon of dynamic space-time, and similarly for the natural, engineering and life sciences. The two simplest Christoffel connections in a spherically symmetric space-time are {2-18}:

$$\Gamma^2_{12} = \Gamma^3_{13} = \frac{1}{r} \quad - (37)$$

and these are similar to the connection (33) of the Coulomb law in ECE theory. For any spherically symmetric space-time the non-vanishing Christoffel connections are:

$$\begin{aligned} \Gamma^2_{12} = \Gamma^3_{13} = 1/r, \quad \Gamma^0_{00} = \frac{1}{c} \frac{\partial d}{\partial t}, \quad \Gamma^0_{01} = \frac{\partial d}{\partial x}, \quad \Gamma^1_{01} = \frac{1}{c} \frac{\partial \beta}{\partial t}, \\ \Gamma^1_{11} = \partial \beta / \partial x, \quad \Gamma^0_{11} = e^{2(\beta-d)} \frac{1}{c} \frac{\partial \beta}{\partial t}, \quad \Gamma^1_{22} = -r e^{-2\beta}, \\ \Gamma^1_{33} = \Gamma^2_{33} = \Gamma^3_{23} = f(\theta), \\ ds^2 = -e^{2d} dt^2 + e^{2\beta} dr^2 + r^2 d\Omega^2. \end{aligned} \quad - (38)$$

In the particular case of the Schwarzschild metric {2-18}:

$$e^{2d} = \left(1 - \frac{2GM}{c^2 r} \right), \quad - (39)$$

$$e^{2\beta} = \left(1 - \frac{2GM}{c^2 r} \right)^{-1}, \quad - (40)$$

and in spherical polar coordinates:

$$\Gamma^0_{01} = d_1 \alpha, \quad \Gamma^0_{11} = e^{2(\beta-d)} d_0 \beta, \quad - (41)$$

$$\Gamma^1_{00} = e^{2(d-\beta)} d_1 \alpha, \quad \Gamma^1_{11} = d_1 \beta.$$

However, the ECE Coulomb law is derived from the Cartan torsion, while the Christoffel connections are for the Cartan curvature in the absence of Cartan torsion.

These considerations of the Coulomb law of generally covariant electro-statics can be extended as follows to the generally covariant Ampere Law of magneto-statics. In ECE theory the magnetic field is:

$$\underline{B}^a = \underline{\nabla} \times \underline{A}^a - \underline{\omega}^a_b \times \underline{A}^b \quad - (42)$$

and the Ampere Law of magneto-statics takes on a generally covariant form as follows:

$$\underline{\nabla} \times \underline{B}^a = \frac{\mu_0}{c} \underline{J}^a. \quad - (43)$$

The magnetic field in general relativity must always be defined by eq. (42) with a non-zero spin connection. The latter is always present in general relativity. In magneto-statics we are dealing with rotational motion, so eq. (42) may be written as:

$$\underline{B}^a = \underline{\nabla} \times \underline{A}^a + g \underline{A}^b \times \underline{A}^c \quad - (44)$$

where the parameter g is defined as {2-18}:

$$g = \frac{\kappa}{A^{(0)}}. \quad - (45)$$

For rotational motion the spin connection is dual to the tetrad if it is assumed that the

electromagnetic and gravitational fields are independent. In Eq. (45) $A^{(o)}$ is a magnitude and

κ has the units of inverse meters. Eq. (44) is therefore:

$$\underline{B}^a = \underline{\nabla} \times \underline{A}^a + \underline{\omega}^b \times \underline{A}^c \quad - (46)$$

where

$$\underline{\omega}^b = \frac{\kappa}{A^{(o)}} \underline{A}^b \quad - (47)$$

If gravitation and electromagnetism are inter-dependent, the general equation (42) must be used, and it cannot be assumed that $\underline{\omega}^a$ is dual to \underline{A}^c .

If it is assumed that:

$$\underline{\nabla} \times \underline{A}^a = \underline{\omega}^b \times \underline{A}^c \quad - (48)$$

then:

$$\underline{B}^a = 2 \underline{\nabla} \times \underline{A}^a \quad - (49)$$

which is the standard model result for each a. The potential is:

$$\underline{A}_{MH} := 2 \underline{A}_{ECE} \quad - (50)$$

and so:

$$\underline{B} = \underline{\nabla} \times \underline{A}_{MH} \quad - (51)$$

as is usual in the standard model {2-18}. So ECE reduces to the standard model of magneto-statics provided the spin connection obeys eq. (48). This is an important result, because in the vast majority of experiments since the eighteenth century both the Coulomb and Ampere laws hold to very high accuracy. So ECE theory must be able to reduce to these

well known results. So both laws are now understood to be very precise laws of general relativity (ECE theory) and not special relativity (Maxwell Heaviside theory). The key advance is that the ECE theory is a generally covariant unified field theory that enables electro-statics and magneto-statics to be unified with all other fields, notably the gravitational field.

The indices a, b and c in eq. (42) originate {2-18} in the tangent space of Cartan geometry, and can be defined in the complex circular basis:

$$a, b, c = (1), (2), (3) \quad - (52)$$

in which the magnetic field is:

$$\begin{aligned} \underline{B}^{(1)*} &= \underline{\nabla} \times \underline{A}^{(1)*} - i\omega^{(2)} \times \underline{A}^{(3)} \\ \underline{B}^{(2)*} &= \underline{\nabla} \times \underline{A}^{(2)*} - i\omega^{(3)} \times \underline{A}^{(1)} \\ \underline{B}^{(3)*} &= \underline{\nabla} \times \underline{A}^{(3)*} - i\omega^{(1)} \times \underline{A}^{(2)}. \end{aligned} \quad - (53)$$

The complex circular basis is defined by the unit vectors:

$$\begin{aligned} \underline{e}^{(1)} &= \frac{1}{\sqrt{2}} (\underline{i} - i\underline{j}) \\ \underline{e}^{(2)} &= \frac{1}{\sqrt{2}} (\underline{i} + i\underline{j}) \\ \underline{e}^{(3)} &= \underline{k} \end{aligned} \quad - (54)$$

with O(3) symmetry:

$$\begin{aligned} \underline{e}^{(1)} \times \underline{e}^{(2)} &= i \underline{e}^{(3)*} \\ \underline{e}^{(2)} \times \underline{e}^{(3)} &= i \underline{e}^{(1)*} \\ \underline{e}^{(3)} \times \underline{e}^{(1)} &= i \underline{e}^{(2)*} \end{aligned} \quad - (55)$$

Here i, j and k are the Cartesian unit vectors. The following are self-consistent vector potential solutions of eq. (53):

$$\underline{A}^{(1)} = \frac{A^{(0)}}{\sqrt{2}} (\underline{i} - i\underline{j}) e^{-ikz} \quad - (56)$$

$$\underline{A}^{(2)} = \underline{A}^{(1)*} = \frac{A^{(0)}}{\sqrt{2}} (\underline{i} + i\underline{j}) e^{i\kappa z}, \quad - (57)$$

$$\underline{A}^{(3)} = A^{(0)} \underline{k} \quad - (58)$$

with vector spin connections:

$$\underline{\omega}^{(1)} = \frac{\omega^{(0)}}{\sqrt{2}} (\underline{i} - i\underline{j}) e^{-i\kappa z}, \quad - (58)$$

$$\underline{\omega}^{(2)} = \frac{\omega^{(0)}}{\sqrt{2}} (\underline{i} + i\underline{j}) e^{i\kappa z}, \quad - (59)$$

$$\underline{\omega}^{(3)} = \omega^{(0)} \underline{k}. \quad - (60)$$

Using the de Moivre Theorem:

$$e^{-i\kappa z} = \cos(\kappa z) - i \sin(\kappa z) \quad - (61)$$

$$e^{i\kappa z} = \cos(\kappa z) + i \sin(\kappa z).$$

Eq. (56) has a real and physical component:

$$\text{Real } \underline{A}^{(1)} = \frac{A^{(0)}}{\sqrt{2}} (\cos(\kappa z) \underline{i} + \sin(\kappa z) \underline{j}) \quad - (62)$$

which is a rotating potential with phase angle:

$$\theta = \kappa z. \quad - (63)$$

It is seen that at $\theta = 0$, $\underline{A}^{(1)}$ is in the \underline{i} axis, and if $\theta = \pi/2$, $\underline{A}^{(1)}$ is in the \underline{j} axis, and so has rotated by 90° . With these definitions it is seen that:

$$\underline{\nabla} \times \underline{A}^{(1)*} = -i \underline{\omega}^{(2)} \times \underline{A}^{(3)} \quad - (64)$$

$$\underline{\nabla} \times \underline{A}^{(2)*} = -i \underline{\omega}^{(3)} \times \underline{A}^{(1)}$$

so the (1) and (2) magnetic fields are:

$$\underline{B}^{(1)} = 2 \underline{\nabla} \times \underline{A}^{(1)}, \quad - (65)$$

$$\underline{B}^{(2)} = 2 \underline{\nabla} \times \underline{A}^{(2)},$$

having the same mathematical form as the standard model. However, general relativity (ECE theory) gives a new result:

$$\underline{B}^{(3)*} = -i \underline{\omega}^{(1)} \times \underline{A}^{(2)} \quad - (66)$$

which does not occur in special relativity. Eqs. (65) may be written as:

$$\underline{B} = \underline{\nabla} \times \underline{A}_{MH}. \quad - (67)$$

From eqs. (64) the magnitude of the spin connection is the wave-number:

$$\omega^{(0)} = \kappa \quad - (68)$$

with units of inverse meters.

(3)

It is important to note the existence of the $\underline{B}^{(3)}$ field in general relativity, eq. (66). In electrodynamics { 2-18 } this is the ECE spin field of electromagnetic radiation. In magneto-statics, to which the Ampere law applies, a magnetic field may also be defined through the spin connection using eq. (66). Using eqs. (56) to (60) the field in eq. (66) is:

$$\underline{B}^{(3)} = B^{(0)} \underline{k} \quad - (69)$$

BUT:

$$\underline{\nabla} \times \underline{A}^{(3)} = \underline{0}. \quad - (70)$$

In electrodynamics the $\underline{B}^{(3)}$ spin field is {2-18}:

$$\underline{B}^{(3)*} = -i \frac{\kappa}{A^{(0)}} \underline{A}^{(1)} \times \underline{A}^{(2)} \quad - (71)$$

where

$$\underline{A}^{(1)} = \underline{A}^{(2)*} = \frac{A^{(0)}}{\sqrt{2}} (\underline{i} - \underline{j}) e^{i(\Omega t - \kappa z)} \quad - (72)$$

where Ω is the electromagnetic angular frequency. In magneto-statics (eq. (56)) the angular frequency Ω is zero. The magneto-static potential rotates (as we have seen), but the electromagnetic potential rotates and also translates along an axis such as Z.

The electrodynamic spin field $\underline{B}^{(3)}$ is observed by its magnetization of matter in the inverse Faraday effect {2-18}. This observation shows that classical electrodynamics and non-linear optics are manifestations of general relativity. The spin connection of the inverse Faraday effect is:

$$\underline{\omega}^{(1)}_{IFE} = \frac{\kappa}{A^{(0)}} \underline{A}^{(1)} \quad - (73)$$

and without the spin connection there is no inverse Faraday effect. Since all physics must be independent of observer influence (must be objective and covariant under the general coordinate transformation), all physics, including electrodynamics, must be general relativity. This means that the electromagnetic field under any circumstance must originate in a spinning space-time described by Cartan torsion {2-18}. In turn this means that the spin connection is non-zero under any circumstance, as emphasized in this section for the electro-statics and magneto-statics. If the spin connection is non-zero the $\underline{B}^{(3)}$ spin field is always non-zero. In Maxwell Heaviside field theory on the other hand the spin connection is zero because the electromagnetic field is philosophically different, it is an entity superimposed on Minkowski space-time, and in this space-time there is no $\underline{B}^{(3)}$ field, contrary to observation.