1)

Some Further Notes on ~~the~~ Reduction of ~~the~~ the Resonance Equation

The initial resonance equation is:

$$\nabla^2\phi - \frac{A}{z}\frac{\partial\phi}{\partial z} + \frac{A}{z^2}\phi = \frac{\rho_o}{\epsilon_o}\cos(kz) \quad -(1)$$

We wish to reduce ~~this~~ this to:

$$\nabla^2\phi - \frac{A}{z_o}\frac{\partial\phi}{\partial z} + \frac{A}{z_o^2}\phi = \frac{\rho_o}{\epsilon_o}\cos(kz) \quad -(2)$$

This can be done by defining:

$$z = z_o e^{\kappa_o x} \quad -(3)$$

so

$$z\frac{d\phi}{dz} = -\frac{1}{\kappa_o}\frac{d\phi}{dx} \quad -(4)$$

$$z^2\frac{d^2\phi}{dz^2} = \frac{1}{\kappa_o}\frac{d\phi}{dx} + \frac{1}{\kappa_o^2}\frac{d^2\phi}{dx^2} \quad -(5)$$

So eq (1) becomes:

$$\frac{d^2\phi}{dx^2} + \kappa_o(A+2)\frac{d\phi}{dx} + A\kappa_o\phi = \frac{\rho_o}{\epsilon_o} z_o^2 e^{2\kappa_o x}\cos(\kappa z)$$

$$= \frac{\rho_o}{\epsilon_o} z_o^2\left(\sinh(2\kappa_o x) + \cosh(2\kappa_o x)\right).$$

$$\cos\left(\kappa z_o\left(\sinh(\kappa_o x) + \cosh(\kappa_o x)\right)\right)$$

$$:= \frac{\rho_o}{\epsilon_o}\cos(\kappa' x) \quad -(6)$$

Now make the change of variable:

2)

$$K_0 \to \frac{1}{Z_0'}, \quad x \to Z \qquad -(7)$$

and :

$$\frac{d^2\phi}{dZ^2} + \frac{(A+2)}{Z_0'}\frac{d\phi}{dZ} + \frac{A}{Z_0'^2}\phi = \frac{\rho_0}{\epsilon_0}\cos(k'Z) \qquad -(8)$$

This is the same form as eq. (2) as required.