1)

Analytical Solution to the Resonance Equation

The resonance equation in Cartesian coordinates is:

$$\nabla^2 \phi - \underline{\omega} \cdot \underline{\nabla} \phi - (\underline{\nabla} \cdot \underline{\omega}) \phi = -\rho / \epsilon_0 \quad -(1)$$

Now we:

$$\underline{\omega} = \frac{A}{Z} \underline{k} \quad -(2)$$

and

$$\rho = -\rho_0 \cos(\kappa Z) \quad -(3)$$

so equation (1) becomes:

$$\boxed{\nabla^2 \phi - \frac{A}{Z} \frac{\partial \phi}{\partial Z} + \frac{A}{Z^2} \phi = \frac{\rho_0}{\epsilon_0} \cos(\kappa Z) \quad -(4)}$$

Compare this with the equation:

$$\ddot{x} + 2\beta \dot{x} + \omega_0^2 x = \alpha \cos \omega t \quad -(5)$$

Frequency resonance from eq. (5) occurs at:

$$\omega_R = (\omega_0^2 - 2\beta^2)^{1/2} \quad -(6)$$

and kinetic energy resonance from eq (5) occurs at

$$\omega_E = \omega_0 \quad -(7)$$

Therefore at some fixed:

$$A / Z = A / Z_0, \quad A/Z^2 = A/Z_0^2$$

2) equation (4) becomes:

$$\nabla^2 \phi - \frac{A}{z_0}\frac{\partial \phi}{\partial z} + \frac{A}{z_0^2}\phi = \frac{\rho_0}{\epsilon_0}\cos(kz) \quad \text{— (9)}$$

<u>Wavenumber</u> resonance occurs for eq. (9) at:

$$\boxed{k_R = \frac{A}{\sqrt{2}}\frac{1}{z_0}} \quad \text{— (10)}$$

and <u>kinetic</u> <u>energy</u> resonance at a wavenumber:

$$\boxed{k_E = \frac{A^{1/2}}{z_0}}. \quad \text{— (11)}$$

At resonance:

$$\boxed{\begin{gathered}\phi_p(k) = \frac{1}{\epsilon_0}\frac{\rho_0\cos(kz-\delta)}{\left(\left(\frac{A}{z_0^2}-k^2\right)^2+\frac{A^2}{z_0^2}k^2\right)^{1/2}} \\ \text{where: } \quad \delta = \tan^{-1}\left(\frac{Ak/z_0}{k^2 - A/z_0^2}\right).\end{gathered}}$$

(12)