Paper 61: Reduction of ECE Theory to the Standard Ampère Law.

The magnetic field in general relativity (ECE theory) is always:

$$\underline{B}^a = \underline{\nabla} \times \underline{A}^a - \underline{\omega}^a{}_b \times \underline{A}^b \quad -(1)$$

and the Ampère Law of magnetostatics is:

$$\underline{\nabla} \times \underline{B}^a = \frac{\mu_0}{c} \tilde{\underline{J}}^a . \quad -(2)$$

The magnetic field in special relativity (Maxwell Heaviside theory) is:

$$\underline{B} = \underline{\nabla} \times \underline{A} \quad -(3)$$

and the Ampère law of special relativity is

$$\underline{\nabla} \times \underline{B} = \frac{\mu_0}{c} \tilde{\underline{J}} . \quad -(4)$$

For each index a eq. (2) reduces to eq. (4).
For each index a eq. (1) is:

$$\underline{B} = \underline{\nabla} \times \underline{A} - \underline{\omega}_b \times \underline{A}^b \quad -(5)$$

where $b = X, Y, Z$. $-(6)$

For simplicity of demonstration only, assume that $\underline{\omega}$ is one dimensional, i.e. is aligned in a given Cartesian axis.

For example:

$$\underline{\omega} = \omega_x \underline{i} \qquad -(7)$$

Then eq. (5) reduces to:

$$\underline{B} = \underline{\nabla} \times \underline{A} - \underline{\omega} \times \underline{A} \qquad -(8)$$

If:

$$\underline{\nabla} \times \underline{A} = -\underline{\omega} \times \underline{A} \qquad -(9)$$

then eq. (1) becomes:

$$\underline{B} = 2 \underline{\nabla} \times \underline{A} \qquad -(10)$$

Doubling the value of $\underline{A}$ makes no difference to the Ampère Law. For example, if:

$$\underline{A} = \frac{B^{(0)}}{2} \left(-Y \underline{i} + X \underline{k} \right) \qquad -(11)$$

$$\underline{B} = B^{(0)} \underline{k} = \underline{\nabla} \times \underline{A} \qquad -(12)$$

and if:

$$\underline{A} = B^{(0)} \left(-Y \underline{i} + X \underline{k} \right) \qquad -(13)$$

$$\underline{B} = 2 B^{(0)} \underline{k} \qquad -(14)$$

In both cases:

$$\underline{\nabla} \times \underline{B} = \frac{\mu_0}{c} \underline{\tilde{J}} \qquad -(15)$$

From eq. (9) the same result is obtained

3) as for electrostatics:

$$\underline{\omega} \rightarrow -\underline{\nabla}. \quad -(16)$$

The spin connection also takes on the same form $(1/r)$. This can be shown as follows. The Z component of eq. (9) is:

$$\left(\frac{\partial A_Y}{\partial X} - \frac{\partial A_X}{\partial Y}\right)\underline{k} = -(\omega_X A_Y - \omega_Y A_X)\underline{k} \quad -(17)$$

Possible solutions are:

$$\frac{\partial A_Y}{\partial X} = -\omega_X A_Y, \quad \frac{\partial A_X}{\partial Y} = -\omega_Y A_X. \quad -(18)$$

From eq. (11):

$$\frac{\partial A_Y}{\partial X} = \frac{\partial A_X}{\partial Y} = \frac{1}{2}, \quad A_Y = \frac{X}{2}, \quad A_X = -\frac{Y}{2}$$

Thus in this example:

$$\omega_X = -\frac{1}{X}, \quad \omega_Y = -\frac{1}{Y} \quad -(19)$$

Since we have chosen eq (7) we have:

$$\boxed{\omega_X = -\frac{1}{X}} \quad -(20)$$

Therefore the spin connection is inverse distance in both electrostatics and magnetostatics. This is an important result which gives great confidence in ECE theory.

4) <u>Comments</u>

The magnetic field in general relativity must always be defined by equation (1), with a non-zero spin connection. The latter is always present in both electrostatics and magnetostatics. In the latter subject we are dealing with rotational motion, so eq. (1) may be re-written as:

$$\left.\begin{aligned}\underline{B}^a &= \underline{\nabla} \times \underline{A}^a - g\,\underline{A}^c \times \underline{A}^b \\ &= \underline{\nabla} \times \underline{A}^a + g\,\underline{A}^b \times \underline{A}^c\end{aligned}\right\} \quad -(21)$$

because for rotational motion the spin connection is dual to the tetrad. This has assumed no interaction between gravitation and electromagnetism which is the assumption of standard model magnetostatics. The factor g in eq. (21) is:

$$g = \frac{\kappa}{A^{(0)}} \quad -(22)$$

where $A^{(0)}$ is a magnitude and κ has the units of inverse metres. Thus eq. (21) may be written as:

$$\boxed{\underline{B}^a = \underline{\nabla} \times \underline{A}^a + \underline{\omega}^b \times \underline{A}^c} \quad -(23)$$

where:

$$\boxed{\underline{\omega}^b = \frac{\kappa}{A^{(0)}} \underline{A}^b} \quad -(24)$$

This is the most self-consistent method of defining a static magnetic field in general relativity if it is assumed that gravitation has no influence on magnetostatics. If this assumption is not made, the general definition (1) must be used, where $\underline{\omega}^a{}_b$ is not dual to $\underline{A}^c$ in general.

If in eq. (23):

$$\underline{\nabla} \times \underline{A}^a = \underline{\omega}^b \times \underline{A}^c \quad -(25)$$

then:

$$\underline{B}^a = 2 \underline{\nabla} \times \underline{A}^a \quad -(26)$$

which is the standard model result for each a. The standard model potential is:

$$\boxed{\underline{A}_{MH} = 2 \underline{A}_{ECE}} \quad -(27)$$

so:

$$\underline{B} = \underline{\nabla} \times \underline{A}_{MH} \quad -(28)$$

as usual.

6) These are important results, because in the vast majority of experiments since the eighteenth century, both the Coulomb and Ampère laws have been found experimentally to hold to great precision. So both laws are now understood as laws of general relativity as required by objectivity in physics. The key difference is that the ECE generally covariant unified field theory allows these laws to be unified with the rest of physics, notably gravitational theory.

Eq. (23) may be simplified to:

$$\underline{B} = \underline{\nabla} \times \underline{A} + \underline{\omega} \times \underline{A} \qquad -(29)$$

provided that the interpretation of a, b, and c in eq. (23) is self-consistent. From the basic Cartan geometry eq. (23) is obtained by considering one frame rotating with respect to another. The tangent indices a, b and c are:

$$a, b, c = (1), (2), (3) \qquad -(30)$$

of the complex circular basis. The components

7) If the vectors in eq. (23) are the usual Cartesian components. So eq. (23) must always be interpreted as:

$$\boxed{\begin{aligned}\underline{B}^{(1)*} &= \underline{\nabla} \times \underline{A}^{(1)*} - i\underline{\omega}^{(2)} \times \underline{A}^{(3)} \\ \underline{B}^{(2)*} &= \underline{\nabla} \times \underline{A}^{(2)*} - i\underline{\omega}^{(3)} \times \underline{A}^{(1)} \\ \underline{B}^{(3)*} &= \underline{\nabla} \times \underline{A}^{(3)*} - i\underline{\omega}^{(1)} \times \underline{A}^{(2)}.\end{aligned}} \quad -(31)$$

The complex circular basis is defined by:

$$\left.\begin{aligned}\underline{e}^{(1)} &= \frac{1}{\sqrt{2}}(\underline{i} - i\underline{j}) \\ \underline{e}^{(2)} &= \frac{1}{\sqrt{2}}(\underline{i} + i\underline{j}) \\ \underline{e}^{(3)} &= \underline{k}\end{aligned}\right\} \quad -(32)$$

so:

$$\left.\begin{aligned}\underline{e}^{(1)} \times \underline{e}^{(2)} &= i\underline{e}^{(3)*} \\ \underline{e}^{(2)} \times \underline{e}^{(3)} &= i\underline{e}^{(1)*} \\ \underline{e}^{(3)} \times \underline{e}^{(1)} &= i\underline{e}^{(2)*}\end{aligned}\right\} \quad -(33)$$

This has $O(3)$ symmetry. Eqs. (31) also have $O(3)$ symmetry.

Now we may choose:

$$\underline{A}^{(1)} = \frac{1}{\sqrt{2}} A^{(0)} (\underline{i} - i\underline{j}) e^{-ikZ} \quad -(34)$$

8)

$$\underline{A}^{(2)} = \underline{A}^{(1)*} = \frac{A^{(0)}}{\sqrt{2}} (\underline{i} + i\underline{j}) e^{ikZ}, \quad -(35)$$

$$\underline{A}^{(3)} = A^{(0)} \underline{k}. \quad -(36)$$

The spin connection vectors are:

$$\underline{\omega}^{(1)} = \frac{\omega^{(0)}}{\sqrt{2}} (\underline{i} - i\underline{j}) e^{-ikZ}, \quad -(37)$$

$$\underline{\omega}^{(2)} = \frac{\omega^{(0)}}{\sqrt{2}} (\underline{i} + i\underline{j}) e^{ikZ} \quad -(38)$$

$$\underline{\omega}^{(3)} = \omega^{(0)} \underline{k}. \quad -(39)$$

With the de Moivre Theorem:

$$\left.\begin{aligned} e^{-ikZ} &= \cos(kZ) - i\sin(kZ) \\ e^{ikZ} &= \cos(kZ) + i\sin(kZ) \end{aligned}\right\} -(40)$$

eq. (34) is:

$$\mathrm{Re}\,\underline{A}^{(1)} = \frac{1}{\sqrt{2}} A^{(0)} \left(\cos(kZ)\,\underline{i} + \sin(kZ)\,\underline{j} \right) \quad -(41)$$

which is a rotating potential, with phase angle:

$$\theta = kZ. \quad -(42)$$

It is seen that if $\theta = 0$, $\underline{A}^{(1)}$ is in the $\underline{i}$ axis, and if $\theta = \pi/2$, $\underline{A}^{(1)}$ is in the $\underline{j}$ axis,

9) so has rotated by 90°. With this definition it is seen that:

$$\left.\begin{aligned} \underline{\nabla} \times \underline{A}^{(1)*} &= -i\,\underline{\omega}^{(2)} \times \underline{A}^{(3)} \\ \underline{\nabla} \times \underline{A}^{(2)*} &= -i\,\underline{\omega}^{(3)} \times \underline{A}^{(1)} \end{aligned}\right\} \quad -(43)$$

so:

$$\left.\begin{aligned} \underline{B}^{(1)} &= 2\,\underline{\nabla} \times \underline{A}^{(1)} \\ \underline{B}^{(2)} &= 2\,\underline{\nabla} \times \underline{A}^{(2)} \end{aligned}\right\} \quad -(44)$$

and the standard model result is regained. However, general relativity gives the new result:

$$\boxed{\underline{B}^{(3)*} = -i\,\underline{\omega}^{(1)} \times \underline{A}^{(2)}} \quad -(45)$$

which does not occur in special relativity. We may write eqns (44) simply as:

$$\underline{B} = \underline{\nabla} \times \underline{A}_{MH} \quad -(46)$$

Proof of Eqns. (43)

i) $\underline{\nabla} \times \underline{A}^{(1)*} = \underline{\nabla} \times \underline{A}^{(2)}$

$$= \frac{A^{(0)}}{\sqrt{2}} \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ e^{ikz} & ie^{ikz} & 0 \end{vmatrix}$$

$$= \frac{A^{(0)}\kappa}{} e^{ikz} \left(\underline{i} + i\,\underline{j} \right) \quad -(47)$$

Thus

$$\boxed{\underline{B}^{(2)} = \underline{\nabla} \times \underline{A}^{(2)}} \quad — (48)$$

wf:

$$B^{(0)} = \kappa A^{(0)} \quad — (49)$$

ii)

$$-i\underline{\omega}^{(2)} \times \underline{A}^{(3)} = -\frac{i\omega^{(0)}A^{(0)}}{\sqrt{2}} \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 1 & i & 0 \\ 0 & 0 & 1 \end{vmatrix} e^{i\kappa Z}$$

$$= -\frac{i\omega^{(0)}A^{(0)}}{\sqrt{2}} \left(i\underline{i} - \underline{j} \right) e^{i\kappa Z}$$

$$= \frac{\omega^{(0)}A^{(0)}}{\sqrt{2}} \left(\underline{i} + i\underline{j} \right) e^{i\kappa Z} \quad — (50)$$

Eqs. (47) and (50) are the same provided ~~that~~:

$$\boxed{\omega^{(0)} = \kappa} \quad — (51)$$

So it is seen that the magnitude of the spin connection is a wave-number κ, with the units of inverse metres. This is self-consistent with the fact that the units of spin connection are inverse metres.

It is important to note the existence of the $\underline{B}^{(3)}$ field in eq. (45). This is a result similar to

11) general relativity. In electrodynamics it is the ECE spin field of electromagnetic radiation. In magnetostatics the magnetic field may also be defined in this way. It is seen in eq. (45) that:

$$\underline{B}^{(3)*} = \underline{B}^{(3)} = -\frac{i\omega^{(0)}A^{(0)}}{2}\begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 1 & -i & 0 \\ 1 & i & 0 \end{vmatrix}$$

$$= \omega^{(0)}A^{(0)}\underline{k}. \quad -(52)$$

Using eq. (51) we obtain:

$$\boxed{\underline{B}^{(3)} = B^{(0)}\underline{k}} \quad -(53)$$

but

$$\underline{\nabla} \times \underline{A}^{(3)} = \underline{0}. \quad -(54)$$

In electrodynamics, the $\underline{B}^{(3)}$ field is defined by:

$$\underline{B}^{(3)*} = -\frac{i\kappa}{A^{(0)}}\underline{A}^{(1)} \times \underline{A}^{(2)} \quad -(55)$$

where:

$$\underline{A}^{(1)} = \underline{A}^{(2)*} = \frac{A^{(0)}}{\sqrt{2}}\left(\underline{i} - i\underline{j}\right)e^{i(\Omega t - \kappa Z)} \quad -(56)$$

where Ω is the electromagnetic angular frequency.

12) The $\underline{B}^{(3)}$ field of electrodynamics is observed as the magnetization of matter in the inverse Faraday effect. This observation shows that electrodynamics is general relativity. The spin connection of the inverse Faraday effect is:

$$\underline{\omega}^{(1)}_{IFE} = \kappa \frac{\underline{A}^{(1)}}{A^{(0)}} \qquad -(57)$$

and without the spin connection there is no inverse Faraday effect. Since all physics must be objective, all physics must be general relativity, and so ~~this~~ is the generally covariant explanation of the inverse Faraday effect required by philosophy.

This argument can be extended to the whole of physics, notably non-linear optics. The next stage of these notes is to extend the argument to electrodynamics from electrostatics and magnetostatics.