1) 60(4). Magnetostatics in Generally Covariant Unified Field Theory, Resonance Effect, Gravito-magnetics

The ECE magnetostatic equations are:

$$\boxed{\begin{aligned}\underline{B}^a &= \underline{\nabla} \times \underline{A}^a - \underline{\omega}^a{}_b \times \underline{A}^b \\ \underline{\nabla} \cdot \underline{B}^a &= \mu_0 \tilde{j}^{0a} \\ \underline{\nabla} \times \underline{B}^a &= \frac{\mu_0}{c} \tilde{\underline{J}}^a\end{aligned}} \quad \begin{aligned}&-(1)\\&-(2)\\&-(3)\end{aligned}$$

i.e.

$$\underline{\nabla} \cdot \left(\underline{\omega}^a{}_b \times \underline{A}^b \right) = -\mu_0 \tilde{j}^{a0} \quad -(4)$$

$$\underline{\nabla} \times \left(\underline{\nabla} \times \underline{A}^a - \underline{\omega}^a{}_b \times \underline{A}^b \right) = \frac{\mu_0}{c} \tilde{\underline{J}}^a \quad -(5)$$

Eq. (4) is the generally covariant Gauss law and Eq. (5) is the generally covariant Ampère Law. In these equations:

$$\underline{\omega}^a{}_b \times \underline{A}^b = \underline{\omega}^1{}_1 \times \underline{A}^1 + \underline{\omega}^1{}_2 \times \underline{A}^2 + \underline{\omega}^1{}_3 \times \underline{A}^3 \quad -(6)$$

If:

$$\tilde{j}^{a0} := \tilde{j}^{a0}_1 + \tilde{j}^{a0}_2 + \tilde{j}^{a0}_3 \quad -(7)$$

equation (4) for example splits into three equations:

$$\underline{\nabla} \cdot \left(\underline{\omega}^1{}_1 \times \underline{A}^1 \right) = -\mu_0 \tilde{j}^{10}_1 \quad -(8)$$

$$\underline{\nabla} \cdot \left(\underline{\omega}^1{}_2 \times \underline{A}^2 \right) = -\mu_0 \tilde{j}^{10}_2 \quad -(9)$$

$$\underline{\nabla} \cdot \left(\underline{\omega}^1{}_3 \times \underline{A}^3 \right) = -\mu_0 \tilde{j}^{10}_3 \quad -(10)$$

2)

So the notation can be simplified by considering each equation to be of the form:

$$\underline{\nabla} \cdot (\underline{\omega} \times \underline{A}) = -\mu_0 \tilde{j}^{\circ} \quad -(11)$$

i.e.

$$\boxed{\underline{\omega} \times \underline{A} = -\mu_0 \int \tilde{j}^{\circ} \, d\underline{r}} \quad -(12)$$

- Similarly, eq. (5) can be split into three equations of the form:

$$\underline{\nabla} \times (\underline{\nabla} \times \underline{A} - \underline{\omega} \times \underline{A}) = \frac{\mu_0}{c} \underline{\tilde{J}} \quad -(13)$$

so:

$$\boxed{\underline{\nabla} \times (\underline{\nabla} \times \underline{A}) = \frac{\mu_0}{c} \underline{\tilde{J}} - \mu_0 \underline{\nabla} \times \int \tilde{j}^{\circ} \, d\underline{r}} \quad -(14)$$

- Using vector identities, eq. (13) is:

$$\boxed{\underline{\nabla}(\underline{\nabla} \cdot \underline{A}) - \nabla^2 \underline{A} - \underline{\omega}(\underline{\nabla} \cdot \underline{A}) + (\underline{\omega} \cdot \underline{\nabla})\underline{A} + (\underline{\nabla} \cdot \underline{\omega})\underline{A} - (\underline{A} \cdot \underline{\nabla})\underline{\omega} = \frac{\mu_0}{c} \underline{\tilde{J}}} \quad -(15)$$

Therefore $\underline{A}$ is amplified at resonance by the current density $\underline{\tilde{J}}$.

3) Consider the Z component of $\tilde{J}$:

$$\frac{\partial^2 A_X}{\partial Z \partial X} + \frac{\partial^2 A_Y}{\partial Z \partial Y} + \omega_X \frac{\partial A_Z}{\partial X} + \omega_Y \frac{\partial A_Z}{\partial Y} - \omega_Z \frac{\partial A_X}{\partial X} - \omega_Z \frac{\partial A_Y}{\partial Y}$$
$$+ \left(\frac{\partial \omega_X}{\partial X} + \frac{\partial \omega_Y}{\partial Y} \right) A_Z - \frac{\partial \omega_Z}{\partial X} A_X - \frac{\partial \omega_Z}{\partial Y} A_Y = \frac{\mu_0}{c} \tilde{J}_Z$$

— (16)

Finally assume that only A_X is non-zero:

$$\boxed{\frac{\partial}{\partial Z}\left(\frac{\partial A_X}{\partial X}\right) - \omega_Z \frac{\partial A_X}{\partial X} - \left(\frac{\partial \omega_Z}{\partial X}\right) A_X = \frac{\mu_0}{c} \tilde{J}_Z}$$

— (17)

This is a relatively simple resonance equation producing a surge of A_X at resonance. Here A_X must be a function of both X and Z and ω_Z must be a function of X.

So at resonance, amplification of the magnetic field occurs by tuning the spin connection. Gravito-magnetic effects are described by eq. (14) via $\tilde{j}^0$, and by eq. (17) via $\tilde{J}_Z$.

Gravito-magnetic effects are amplified at resonance