THE EFFECT OF TORSION ON THE SCHWARZSCHILD METRIC AND LIGHT DEFLECTION DUE TO GRAVITATION.

by

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ABSTRACT

The effect of torsion on the Schwarzschild metric and light deflection due to gravitation is calculated straightforwardly using the tetrad method at the root of Einstein Cartan Evans (ECE) unified field theory. Consideration of torsion changes several of the assumptions at the root of standard model cosmologies such as Big Bang, and torsion is shown to affect the deflection of light due to gravitation. Thus, any deviations from Einstein Hilbert theory may be explained by the presence of torsion.

Keywords: Einstein Cartan Evans (ECE) unified field theory, Schwarzschild metric, light deflection due to gravitation, effect of torsion on standard model cosmologies.

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1. INTRODUCTION

Light deflection due to gravitation is a famous prediction of gravitational general relativity, and is based on the Einstein Hilbert (EH) field equation published independently by Einstein and Hilbert in 1916 as is well known. The phenomenon of light deflection by the sun can now be measured to an accuracy of one part in one hundred thousand (NASA Cassini) and even more accurate tests are being prepared by NASA. It is shown in Section 3 that any small deviations from the EH result that may become observable can be understood straightforwardly as being due to space-time torsion in general relativity. The Cartan torsion is of key importance to the recently inferred {1-16} Einstein Cartan Evans (ECE) unified field theory because the electromagnetic field is Cartan torsion within a factor $cA^{(0)}$ with the units of volts and thus referred to as the primordial voltage. The EH equation is well known to produce twice the Newtonian result for the deflection angle of light grazing a mass, such as the mass of the sun. In Section 2 this result is derived straightforwardly using the tetrads appropriate to the Schwarzschild metric (SM). The latter was used in the original and famous test by Eddington and co-workers and is used here to illustrate the effect of torsion. More generally in ECE field theory metrics must be calculated in the presence of Cartan torsion, which changes many of the basic assumptions of standard model cosmology. In the presence of Cartan torsion the Ricci cyclic equation is no longer true, the Riemann tensor is no longer anti-symmetric in its first two indices, the symmetric metric and symmetric Ricci tensor are true only if the central part of torsion affected motion is considered, and the symmetric Christoffel symbol must be replaced by a more general and asymmetric gamma connection. The neglect of Cartan torsion in cosmologies such as Big Bang is arbitrary. Without Cartan torsion the gravitational field cannot be unified with the electromagnetic field, which as originally inferred by Cartan himself, is the Cartan torsion within $cA^{(0)}$ {1-16}. Attempts to interpret astronomical data in terms of a purely central cosmology such as Big Bang are therefore purposeless because torsion is likely to pervade all cosmologies. There is no reason to assert that Cartan curvature is always large in magnitude in comparison with Cartan torsion. This EH assumption appears to be true for the sun, but may not be true for other cosmological objects.

2. CALCULATION OF GRAVITATIONAL LIGHT DEFLECTION USING THE TETRAD METHOD.

The SM is well known to be the first solution to the Einstein Hilbert field equation, and was inferred in 1916. The SM metric is the static solution for a spherically symmetric space-time and produces a deflection of light twice that expected from Newtonian theory. For light deflection from the sun this result of the SM has been verified by NASA Cassini to one part in one hundred thousand. So for the sun, EH theory is adequate to this accuracy. For other systems however, this may not be the case at all, because there is no reason to assume that Cartan torsion is small in magnitude compared with Cartan curvature for all cosmological objects {1-16}. The SM $g_{\mu\nu}$ is necessarily symmetric in its indices:

$$g_{\mu\nu} = g_{\nu\mu} \quad -(1)$$

and defines the square of the line element:

$$ds^2 = g_{\mu\nu} \, dx^\mu dx^\nu \quad -(2)$$

where x^μ is the four-coordinate:

$$x^\mu = (ct, x, y, z). \quad -(3)$$

This symmetric metric is defined in terms of the tetrad of ECE theory {1-16} by:

$$g_{\mu\nu} = q^a_\mu \, q^b_\nu \, \eta_{ab} \quad -(4)$$

where η_{ab} is the Minkowski metric of flat space-time. The latter is defined by:

$$\eta_{ab} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} . \qquad -(5)$$

In spherical polar coordinates the line element is:

$$ds^2 = -c^2 dt^2 + dr^2 + r^2 d\Omega^2 \qquad -(6)$$

where

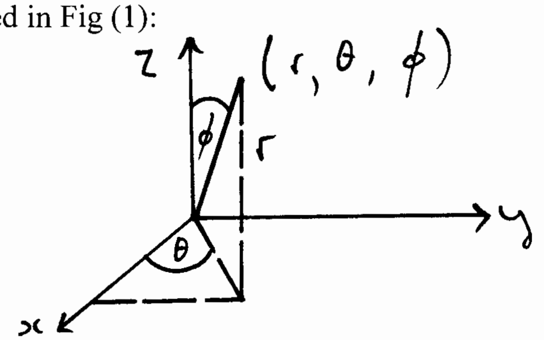
$$d\Omega^2 = d\theta^2 + \sin^2\theta \, d\phi^2 \qquad -(7)$$

and the SM in spherical polar coordinates and complete S.I. units is well known to be:

$$ds^2 = -\left(1 - \frac{2GM}{c^2 r}\right) c^2 dt^2 + \left(1 - \frac{2GM}{c^2 r}\right)^{-1} dr^2 + r^2 d\Omega^2 . \qquad -(8)$$

Here G is the Newton gravitational constant, M is the mass of the object responsible for the light deflection (e.g. the sun), c is the speed of light and where r is the radial coordinate of the spherical polar system defined in Fig (1):

Fig. (1)



The SM reduces to the Minkowski result in the limit of large r or small M as is well known.

In Cartesian coordinates the Minkowski metric is found from:

$$ds^2 = -c^2 dt^2 + dx^2 + dy^2 + dz^2 \qquad -(9)$$

and in spherical polar coordinates it is:

$$\eta_{ab} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2 \sin^2\phi \end{bmatrix} . \quad -(10)$$

The SM in spherical polar coordinates is:

$$g_{\mu\nu} = \begin{bmatrix} -\left(1 - \frac{2GM}{c^2 r}\right) & 0 & 0 & 0 \\ 0 & \left(1 - \frac{2GM}{c^2 r}\right)^{-1} & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2 \sin^2\phi \end{bmatrix} . \quad -(11)$$

Therefore from a comparison of the diagonal elements in Eqs. (10) and (11) the tetrads of the SM may be found straightforwardly. The non-zero Minkowski elements in spherical polar coordinates are:

$$\eta_{00} = -1, \; \eta_{11} = 1, \; \eta_{22} = r^2, \; \eta_{33} = r^2 \sin^2\phi \quad -(12)$$

and the non-zero SM elements in the same coordinates are:

$$g_{00} = -\left(1 - \frac{2GM}{c^2 r}\right), \; g_{11} = \left(1 - \frac{2GM}{c^2 r}\right)^{-1}, \; g_{22} = r^2, \; g_{33} = r^2 \sin^2\phi \quad -(13)$$

where in general:

$$g_{00} = q^a_{\ 0} q^b_{\ 0} \eta_{ab} \quad -(14)$$

$$\vdots$$

$$g_{33} = q^a_{\ 3} q^b_{\ 3} \eta_{ab} \quad -(15)$$

Considering only the diagonal elements Eqs. (14) to (15) simplify to:

$$g_{00} = q^0_{\ 0} q^0_{\ 0} \eta_{00} \quad -(16)$$

$$\vdots$$

$$g_{33} = q^3_{\ 3} q^3_{\ 3} \eta_{33} \quad -(17)$$

Therefore the required tetrad elements of the SM are:

$$q^0_{\ 0} = \left(1 - \frac{2GM}{c^2 r}\right)^{1/2} \quad -(18)$$

$$q^1_{\ 1} = \left(1 - \frac{2GM}{c^2 r}\right)^{-1/2} \quad -(19)$$

$$q^2_{\ 2} = 1 \quad -(20)$$

$$q^3_{\ 3} = 1. \quad -(21)$$

In the limit of large r or small M these reduce to the correct Minkowski elements:

$$g_{00} \rightarrow \eta_{00} \ \text{etc.} \quad -(22)$$

so Eq. (16) -(17) are correctly compatible with this limit. According to the ECE Lemma {1-16}

$$\square q^0_{\ 0} = R_0 q^0_{\ 0} \quad -(23)$$

$$\square q^1_{\ 1} = R_1 q^1_{\ 1} \quad -(24)$$

so scalar curvatures R_0 and R_1 are generated by two of the tetrad elements of the SM.

There are no ECE scalar curvatures produced by the Minkowski metric, and this result is compatible with the fact that that metric describes a flat space-time with no curvature. The four tetrads of the Minkowski metric are all unity. In spherical polar coordinates:

$$r = \left(x^2 + y^2 + z^2\right)^{1/2} \quad -(25)$$

so Eqs. (23) and (24) reduce to:

$$\nabla^2 q^0_{\ 0} = -R_0 q^0_{\ 0} \quad -(26)$$

$$\nabla^2 q^1_{\ 1} = -R_1 q^1_{\ 1} \quad -(27)$$

compatible with the fact that the SM is a static solution of the EH field equation for a spherically symmetric spacetime.

The spherical polar coordinates and Cartesian coordinates are related by:

$$\left.\begin{aligned} x &= r\sin\phi\cos\theta \\ y &= r\sin\phi\sin\theta \\ z &= r\cos\phi \end{aligned}\right\} \quad -(28)$$

so:

$$x^2 + y^2 + z^2 = r^2. \quad -(29)$$

The infinitesimal elements are defined {17} by:

$$\left.\begin{aligned} dx &= -r\sin\phi\sin\theta\, d\theta + r\cos\phi\cos\theta\, d\phi + \sin\phi\cos\theta\, dr \\ dy &= r\sin\phi\cos\theta\, d\theta + r\cos\phi\sin\theta\, d\phi + \sin\phi\sin\theta\, dr \\ dz &= -r\sin\phi\, d\phi + \cos\phi\, dr \end{aligned}\right] \quad -(30)$$

so the square of the line element is:

$$ds^2 = dx^2 + dy^2 + dz^2 = dr^2 + r^2 d\phi^2 + r^2\sin^2\phi\, d\theta^2. \quad -(31)$$

The space-like metric elements in curvilinear coordinates are the squares of the scale factors {17}:

$$g_{11} = h_1^2, \quad g_{22} = h_2^2, \quad g_{33} = h_3^2. \quad -(32)$$

The scale factors in spherical polar coordinates {17} are:

$$h_1 = h_r = 1, \quad h_2 = h_\phi = r, \quad h_3 = h_\theta = r\sin\phi \quad -(33)$$

in Euclidean space-time. The surface of a sphere is:

$$S = \int_0^{2\pi} d\theta \int_0^{\pi} r^2 \sin\phi\, d\phi = 4\pi r^2 \quad -(34)$$

and the volume of a sphere is:

$$V = \int_0^r S\,dr = \frac{4}{3}\pi r^3 . \quad -(35)$$

The Euclidean unit vectors of the spherical polar coordinate system are {17}:

$$\left.\begin{aligned} \underline{e}_r &= \sin\phi\cos\theta\,\underline{i} + \sin\phi\sin\theta\,\underline{j} + \cos\phi\,\underline{k} \\ \underline{e}_\phi &= \cos\phi\cos\theta\,\underline{i} + \cos\phi\sin\theta\,\underline{j} - \sin\phi\,\underline{k} \\ \underline{e}_\theta &= -\sin\theta\,\underline{i} + \cos\theta\,\underline{j} \end{aligned}\right] -(36)$$

where $\underline{i}$, $\underline{j}$ and $\underline{k}$ are the unit vectors of the Cartesian system. The Euclidean vector field in spherical polar coordinates is therefore:

$$\begin{aligned} \underline{V} &= V_r\,\underline{e}_r + V_\phi\,\underline{e}_\phi + V_\theta\,\underline{e}_\theta \quad -(37) \\ &= V_x\,\underline{i} + V_y\,\underline{j} + V_z\,\underline{k} . \end{aligned}$$

In Cartan geometry {1-16, 18}, the governing equations of the EH equation and the SM are torsion-less:

$$T^a = d\wedge q^a + \omega^a{}_b \wedge q^b = 0 \quad -(38)$$

$$R^a{}_b = d\wedge\omega^a{}_b + \omega^a{}_c \wedge \omega^c{}_b \quad -(39)$$

$$R^a{}_b \wedge q^b = 0 \quad -(40)$$

$$d\wedge R^a{}_b + \omega^a{}_c \wedge R^c{}_b - R^a{}_c \wedge \omega^c{}_b = 0 . \quad -(41)$$

Here T^a is the Cartan torsion form, q^a is the Cartan tetrad form, $\omega^a{}_b$ is the spin connection, and $R^a{}_b$ is the curvature or Riemann form of Cartan geometry. The elements of the tetrad of the SM are diagonal as shown already, and the non-vanishing elements of the Riemann tensor of the SM are:

$$R^0{}_{101},\ R^0{}_{202},\ R^0{}_{303},\ R^0{}_{212},\ R^0{}_{313},\ R^1{}_{212},\ R^1{}_{313},\ R^2{}_{323} . \quad -(42)$$

The Riemann form and Riemann tensor are related by {1-6, 18}:

$$R^a{}_{b\mu\nu} = q^a_\rho\, q^\sigma{}_b\, R^\rho{}_{\sigma\mu\nu}. \quad -(43)$$

In the presence of the Cartan torsion, equations (38) to (41) become:

$$T^a = d\wedge q^a + \omega^a{}_b \wedge q^b \quad -(44)$$

$$R^a{}_b = d\wedge\omega^a{}_b + \omega^a{}_c \wedge \omega^c{}_b \quad -(45)$$

$$d\wedge T^a + \omega^a{}_b \wedge T^b := R^a{}_b \wedge q^b \quad -(46)$$

$$d\wedge R^a{}_b + \omega^a{}_c \wedge R^c{}_b - R^a{}_c \wedge \omega^c{}_b := 0. \quad -(47)$$

Eqs. (44) and (45) are the two Cartan structure equations, and Eqs. (46) and (47) are the two Bianchi identities. These are well known equations of standard Cartan geometry and form the basis of ECE theory {1-16} through the ansatzen:

$$A^a = A^{(0)} q^a \quad -(48)$$

$$F^a = A^{(0)} T^a \quad -(49)$$

first proposed by Cartan himself in well known correspondence with Einstein. Here A^a is the electromagnetic potential form, and F^a is the electromagnetic field form. In the EH equation and SM there is no consideration given to the interaction of gravitation with other fields such as electromagnetism. In the presence of torsion the familiar Ricci cyclic equation (40) of EH theory and the SM is no longer obeyed. In tensor notation the Ricci cyclic equation is:

$$R_{\sigma\mu\nu\rho} + R_{\sigma\rho\mu\nu} + R_{\sigma\nu\rho\mu} = 0 \quad -(50)$$

but this is not the case in the presence of torsion. The latter means therefore that the Riemann tensor is no longer anti-symmetric in its first two indices, and that the Christoffel connection becomes the general gamma connection no longer symmetric in its lower two indices. Cartan torsion fundamentally changes cosmologies based on the EH equation, for example Big Bang.

Restricting attention in this section to the EH field theory, the spin connection of the SM may be obtained from the tetrad of the SM using:

$$d \wedge q^a + \omega^a{}_b \wedge q^b = 0. \quad -(51)$$

The Riemann form and the spin connection are related by the second Cartan structure equation (45):

$$R^a{}_b = d \wedge \omega^a{}_b + \omega^a{}_c \wedge \omega^c{}_b. \quad -(52)$$

In Section 3 the equation (51) will be perturbed by a small torsion δT^a, to give:

$$d \wedge q^a + \omega^a{}_b \wedge q^b = \delta T^a \quad -(53)$$

while in the rest of Section 2 the light deflection of the SM will be calculated by the tetrad method. This is shown to be much simpler and easier to use and understand than the conventional metric method {18, 19}. Use of the tetrad method also allows the effect of torsion to be calculated via equation (53).

The SM written out in spherical polar coordinates (Fig. (1)) is:

$$ds^2 = -\left(1 - \frac{2GM}{c^2 r}\right) c^2 dt^2 + \left(1 - \frac{2GM}{c^2 r}\right)^{-1} dr^2 + r^2 d\phi^2 + r^2 \sin^2\phi \, d\theta^2. \quad -(54)$$

Light travels along null paths:

$$ds^2 = 0. \quad -(55)$$

Now restrict consideration to a single plane through the center of mass:

$$\theta = 0. \quad -(56)$$

Therefore Eq. (54) becomes:

$$c^2 dt^2 = \left(1 - \frac{2GM}{c^2 r}\right)^{-2} dr^2 + r^2 \left(1 - \frac{2GM}{c^2 r}\right)^{-1} d\phi^2 . \quad -(57)$$

The metric corresponding to this equation is:

$$g_{\mu\nu} = \begin{bmatrix} \left(1 - \frac{2GM}{c^2 r}\right)^{-2} & 0 \\ 0 & r^2 \left(1 - \frac{2GM}{c^2 r}\right)^{-1} \end{bmatrix} \quad -(58)$$

which reduces to the Minkowski metric for large r or small M:

$$\eta_{ab} = \begin{bmatrix} 1 & 0 \\ 0 & r^2 \end{bmatrix} . \quad -(59)$$

Therefore using Eq. (4) the tetrads are:

$$q_{rr} = \left(1 - \frac{2GM}{c^2 r}\right)^{-1} , \quad -(60)$$

$$q_{\phi\phi} = \left(1 - \frac{2GM}{c^2 r}\right)^{-1/2} . \quad -(61)$$

If:

$$c^2 r \gg 2GM \quad -(62)$$

then:

$$q_{rr} \rightarrow 1 + \frac{2GM}{c^2 r} + \ldots \quad (63)$$

$$q_{\phi\phi} \rightarrow 1 + \frac{GM}{c^2 r} + \ldots \quad (64)$$

The tetrad element q_{rr} means that r is not a straight line, it is a curve:

$$\gamma(r) = \gamma^{(0)} q_{rr} \quad -(65)$$

where $\gamma^{(0)}$ is a scalar proportionality factor. By differentiation with respect to r:

$$c^2 \frac{\partial q_{rr}}{\partial r} = -2\frac{GM}{r^2}. \quad -(66)$$

The Newtonian force between a photon of mass m and the sun of mass M is:

$$F = -\frac{GmM}{r^2}. \quad -(67)$$

The force from Eq. (66) is:

$$F = -mc^2 \frac{\partial q_{rr}}{\partial r} = -2G\frac{mM}{r^2}. \quad -(68)$$

This is twice the Newtonian force and is $-\partial q_{rr} / \partial r$ multiplied by the photon rest energy:

$$E_0 = mc^2 = \hbar \omega_0. \quad -(69)$$

Eq. (69) is the Planck / Einstein / de Broglie equation. Using the equivalence of inertial and gravitational mass, the force from Eq. (68) is:

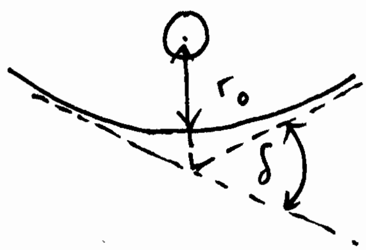
$$F = mg = -mc^2 \frac{\partial q_{rr}}{\partial r} \quad -(70)$$

so the acceleration due to gravity is due to the r derivative of the radial tetrad within a factor c^2 :

$$g = -c^2 \frac{\partial q_{rr}}{\partial r} . \quad -(71)$$

The angle of deflection in the Eddington experiment is defined by Fig. (2):

Fig (2).



The Newtonian result is:

$$\delta(\text{Newton}) = \frac{2MG}{c^2 r_0} \quad -(72)$$

where r_0 is the distance of closest approach. So the result from the EH theory is twice this from Eq. (68):

$$\delta(\text{Schwarzschild}) = \frac{4MG}{c^2 r_0} . \quad -(73)$$

Using the tetrad method the effect of Cartan torsion on this result will be calculated in Section 3. The tetrad method, developed in this Section for the first time, is straightforward, and is ideally suited to calculate the effect of torsion from Eq. (53) from standard Cartan geometry. The metric method of calculating the Eddington deflection is much more complicated.