1)

Notes 58(8): Simple Example of the Effect of Torsion on Deflection Angle

The angle of deflection is given by:

$$\boxed{\delta = 2\left(q_{rr} - 1\right)_{r=r_0}} \quad -(1)$$

In the absence of torsion:

$$d \wedge q_{rr,0} = -\omega_0 \wedge q_{rr,0} \quad -(2)$$

where ω_0 is the spin connection in the absence of torsion.

In the presence of torsion:

$$d \wedge q_{rr,T} = -\omega \wedge q_{rr,T} + \delta T \quad -(3)$$

If δT is considered to be a small perturbation then in a first approximation:

$$\omega \sim \omega_0 \quad -(4)$$

and

$$\omega \wedge q_{rr,T} \sim \omega_0 \wedge q_{rr,0} \quad -(5)$$

so

$$d \wedge q_{rr,T} - d \wedge q_{rr,0} \sim \delta T \quad -(6)$$

or

$$\boxed{d \wedge (\delta q_{rr}) \sim \delta T} \quad -(7)$$

and

$$\boxed{\Delta \delta \sim 2\left(\delta q_{rr}\right)_{r=r_0}} \quad -(8)$$

2)

From eqs. (7) and (8) it is clear that the torsional perturbation δT will change the angle of deflection by $\Delta\delta$. In Cartesian coordinates introduce a perturbation:

$$\delta q_{rr} = \frac{1}{\sqrt{2}}(1-i)e^{i\phi} \quad -(9)$$

$$\sim \frac{1}{\sqrt{2}} \sin\phi \sim \frac{1}{\sqrt{2}}\phi \quad -(10)$$

for $\phi << 1$. Thus:

$$\boxed{\Delta\delta \sim \frac{2}{\sqrt{2}}\phi} \quad -(11)$$

This is a very simple result that illustrates the effect of torsion on the angle of deflection in the Eddington experiment. From experimental data on gravitational lensing it is known that ϕ must be very small for the sun photon system because the EH result is accurate there to one part in 10^5. For other systems ϕ may be larger.

In this very simple example the δq_{rr} is due to a helical spacetime perturbation.