1)

Notes for Paper 56, Part Three

Development of the Geometrical Equations of the Class of Aharonov Bohm Effects.

The class of electromagnetic AB effects is defined by:

$$A^{(0)} D \wedge q = 0 \quad -(1)$$

and the class of gravitational AB effects by:

$$D \wedge \omega = 0. \quad -(2)$$

Here $cA^{(0)}$ is a primordial voltage which is always non-zero. Eq. (1) means that when the electromagnetic field is zero the electromagnetic potential can be non-zero. Eq. (2) means that when the Riemann curvature is zero, the spin connection is non-zero. This is what is observed in the electromagnetic and gravitational AB effects. When the field is excluded experimentally there is still an observable effect of the potential. From eq. (1) the electromagnetic potential in zero-field region is given by:

$$d \wedge A^a + \omega^a{}_b \wedge A^b = 0. \quad -(3)$$

From eq (2) the spin ~~curvature~~ connection in zero-curvature region is given by:

$$d \wedge \omega^a{}_b + \omega^a{}_c \wedge \omega^c{}_b = 0. \quad -(4)$$

In eq. (3): $F^a = 0 \quad -(5)$

2) and in eq. (4):

$$R^a{}_b = 0 \quad -(6)$$

Even though the electromagnetic field is zero, an electron can still interact with the potential A^a defined by eq. (3) through the minimal prescription:

$$p^a_\mu = p^a_\mu + eA^a_\mu \quad -(7)$$

where:

$$p^a_\mu = p^{(0)} q^a_\mu \quad -(8)$$

is the momentum tetrad.

In the standard model this is not possible, because in the standard model:

$$F = d \wedge A \quad -(9)$$

so if F is zero, so is A. In ECE theory A^a can be non-zero if F^a is zero, as observed for example in the Chambers experiment.

The task is to solve eq. (3) for the potential when the field is zero. This potential gives the Aharonov Bohm effect.

In the absence of gravitation:

$$\omega^a{}_b = -\frac{\kappa}{2} \epsilon^a{}_{bc} q^c \quad -(10)$$

(eq. (6.23) of vol. 1 of "Generally Covariant Unified Field Theory" (Abramis 2005)).

3) Here:

$$\epsilon^{a}{}_{bc} = \eta^{ad}\epsilon_{dbc} \quad -(11)$$

where

$$\eta^{ad} = \underset{\text{diag}}{=}(1, -1, -1, -1) \quad -(12)$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \quad -(13)$$

The 3-D antisymmetric tensor in eq. (11) is defined by:

$$\epsilon_{123} = \epsilon_{231} = \epsilon_{312} = 1 \quad -(14)$$

$$\epsilon_{132} = \epsilon_{213} = \epsilon_{321} = -1 \quad -(15)$$

and

$$\epsilon_{012} = \epsilon_{120} = \epsilon_{201} = 1 \quad -(16)$$

$$\epsilon_{021} = \epsilon_{102} = \epsilon_{210} = -1, \quad -(17)$$

all other elements zero.

Therefore:

$$\epsilon^{1}{}_{23} = g^{1d}\epsilon_{d23} = g^{11}\epsilon_{123} = -\epsilon_{123} \quad -(18)$$

$$\epsilon^{3}{}_{12} = g^{3d}\epsilon_{d12} = g^{33}\epsilon_{312} = -\epsilon_{312} \quad -(19)$$

$$\epsilon^{2}{}_{31} = g^{2d}\epsilon_{d31} = g^{22}\epsilon_{231} = -\epsilon_{231} \quad -(20)$$

$$\epsilon^{0}{}_{12} = g^{00}\epsilon_{012} = \epsilon_{012} \quad -(21)$$

$$\epsilon^{1}{}_{20} = g^{11}\epsilon_{120} = -\epsilon_{120} \quad -(22)$$

$$\epsilon^{2}{}_{01} = g^{22}\epsilon_{201} = -\epsilon_{201} \quad -(23)$$

Eq. (3) may be developed as follows:

$$d \wedge A^0 + \omega^0{}_b \wedge A^b = 0 \quad -(24)$$

$$d \wedge A^1 + \omega^1{}_b \wedge A^b = 0 \quad -(25)$$

$$d \wedge A^2 + \omega^2{}_b \wedge A^b = 0 \quad -(26)$$

$$d \wedge A^3 + \omega^3{}_b \wedge A^b = 0. \quad -(27)$$

In these equations there is summation over repeated b indices, and because of antisymmetry the indices of $\omega^a{}_b$ must be different. Therefore:

$$d \wedge A^0 + \omega^0{}_1 \wedge A^1 + \omega^0{}_2 \wedge A^2 + \omega^0{}_3 \wedge A^3 = 0 \quad -(28)$$

$$d \wedge A^1 + \omega^1{}_0 \wedge A^0 + \omega^1{}_2 \wedge A^2 + \omega^1{}_3 \wedge A^3 = 0 \quad -(29)$$

$$d \wedge A^2 + \omega^2{}_0 \wedge A^0 + \omega^2{}_1 \wedge A^1 + \omega^2{}_3 \wedge A^3 = 0 \quad -(30)$$

$$d \wedge A^3 + \omega^3{}_0 \wedge A^0 + \omega^3{}_1 \wedge A^1 + \omega^3{}_2 \wedge A^2 = 0 \quad -(31)$$

These are four equations in four unknowns, because eq. (10) expresses the spin connection in terms of the tetrad and therefore the potential. The spin connections are as follows.

$$\omega^0{}_1 = -\frac{\kappa}{2}\epsilon^0{}_{12}\, q^2 = -\frac{\kappa}{2} q^2 \quad (32)$$

$$\omega^0{}_2 = -\frac{\kappa}{2}\epsilon^0{}_{21}\, q^1 = \frac{\kappa}{2} q^1 \quad (33)$$

$$\omega^0{}_3 = 0 \quad (34)$$

$$\omega^1{}_0 = -\omega^0{}_1 = \frac{\kappa}{2} q^2 \quad (35)$$

$$\omega^2{}_0 = -\omega^0{}_2 = -\frac{\kappa}{2} q^1 \quad (36)$$

$$\omega^3{}_0 = -\omega^0{}_3 = 0 \quad (37)$$

$$\omega^1{}_2 = -\frac{\kappa}{2}\left(\epsilon^1{}_{20}\, q^0 + \epsilon^1{}_{23}\, q^3\right)$$

$$= \frac{\kappa}{2}\left(q^0 + q^3\right) \quad (38)$$

$$\omega^2{}_1 = -\omega^1{}_2 = -\frac{\kappa}{2}\left(q^0 + q^3\right) \quad (39)$$

$$\omega^1{}_3 = -\frac{\kappa}{2}\epsilon^1{}_{32}\, q^2 = -\frac{\kappa}{2} q^2 \quad (40)$$

$$\omega^3{}_1 = -\omega^1{}_3 = \frac{\kappa}{2} q^2 \quad (41)$$

$$\omega^2{}_3 = -\frac{\kappa}{2}\epsilon^2{}_{31}\, q^1 = \frac{\kappa}{2} q^1 \quad (42)$$

$$\omega^3{}_2 = -\omega^2{}_3 = -\frac{\kappa}{2} q^1 \quad (43)$$

We may now substitute back into eqs. (28) to (31) to eliminate the spin connections

) This gives the equations:

$$d \wedge A^0 - \frac{\kappa}{2} q^2 \wedge A^1 + \frac{\kappa}{2} q^1 \wedge A^2 = 0 \quad -(44)$$

$$d \wedge A^1 + \frac{\kappa}{2} q^2 \wedge A^0 + \frac{\kappa}{2} (q^0 + q^3) \wedge A^2 - \frac{\kappa}{2} q^2 \wedge A^3 = 0 \quad -(45)$$

$$d \wedge A^2 - \frac{\kappa}{2} q^1 \wedge A^0 - \frac{\kappa}{2} (q^0 + q^3) \wedge A^1 + \frac{\kappa}{2} q^1 \wedge A^3 = 0 \quad -(46)$$

$$d \wedge A^3 + \frac{\kappa}{2} q^2 \wedge A^1 - \frac{\kappa}{2} q^1 \wedge A^2 = 0. \quad -(47)$$

The tetrad and potential are related by:

$$A^a_\mu = A^{(0)} q^a_\mu \quad -(48)$$

where $cA^{(0)}$ is the primordial voltage.

By antisymmetry of the wedge product:

$$q^2 \wedge A^1 = - q^1 \wedge A^2 \quad -(49)$$

and so on. Therefore eq. (44) becomes:

$$d \wedge A^0 + g A^1 \wedge A^2 = 0, \quad -(50)$$

where

$$g = \frac{\kappa}{A^{(0)}}. \quad -(51)$$

7)

Similarly, eq. (47) becomes:

$$d \wedge A^3 - g A^1 \wedge A^2 = 0 \qquad - (51)$$

and eqs (45) and (46) become:

$$d \wedge A^1 - g A^2 \wedge A^3 = 0 \qquad - (52)$$

and

$$d \wedge A^2 - g A^3 \wedge A^1 = 0. \qquad - (53)$$

It is seen that the space-like potentials in eqns (51) - (53) have $O(3)$ symmetry

$$\boxed{\begin{aligned} d \wedge A^1 &= g A^2 \wedge A^3 \\ d \wedge A^2 &= g A^3 \wedge A^1 \\ d \wedge A^3 &= g A^1 \wedge A^2 \end{aligned}} \qquad \begin{aligned} &- (54) \\ &- (55) \\ &- (56) \end{aligned}$$

From eqs. (50) and (51):

$$\boxed{d \wedge A^0 = - d \wedge A^3 = - g A^1 \wedge A^2} \qquad - (57)$$

Therefore from eq. (57):

$$A^0 = - A^3 \qquad - (58)$$