1)

Notes 56(1)

Geodesics in ECE Theory

In standard general relativity the geodesic is defined as the path that parallel transports its own tangent vector. If $x^\mu(\lambda)$ is any curve, the tangent vector is:

$$x^\mu(\lambda) = \frac{dx^\mu}{d\lambda} \quad -(1)$$

The covariant derivative along the path is:

$$\frac{D}{d\lambda} := \frac{dx^\mu}{d\lambda} D_\mu \quad -(2)$$

where D_μ is the covariant derivative. The parallel transport of any tensor T is then defined as:

$$\left(\frac{DT}{d\lambda}\right)^{\mu_1\mu_2\ldots\mu_k}_{\nu_1\nu_2\ldots\nu_l} = \frac{dx^\sigma}{d\lambda} D_\sigma T^{\mu_1\mu_2\ldots\mu_k}_{\nu_1\nu_2\ldots\nu_l}$$

$$= 0. \quad -(3)$$

In Einstein Hilbert (EH) field theory the connection is metric compatible:

$$D_\mu g_{\rho\nu} = 0. \quad -(4)$$

Such a connection is always parallel transported (Carroll eqn (3.33)):

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$$\frac{D g_{\mu\nu}}{d\lambda} = \frac{dx^\sigma}{d\lambda} D_\sigma g_{\mu\nu} = 0 \quad -(5)$$

The equation of the geodesic is:

$$\frac{D}{d\lambda}\left(\frac{dx^\mu}{d\lambda}\right) = 0 \quad -(6)$$

because the tangent vector is parallel transported

Eq. (6) can be re-written as:

$$\boxed{\frac{d^2 x^\mu}{d\lambda^2} + \Gamma^\mu_{\rho\sigma}\frac{dx^\rho}{d\lambda}\frac{dx^\sigma}{d\lambda} = 0} \quad -(7)$$

This is the well known geodesic equation of EH field theory, in which geodesics are the paths followed by unaccelerated particles. In flat spacetime eq. (7) becomes:

$$\frac{d^2 x^\mu}{d\lambda^2} = 0 \quad -(8)$$

which is the equation of a straight line.

The parameter λ can be related to the proper time τ by:

) An affine parameter is related to the proper time in this way. If:

$$\lambda = \tau \quad -(10)$$

the geodesic equation is:

$$\frac{d^2x^\mu}{d\tau^2} + \Gamma^\mu_{\rho\sigma}\frac{dx^\rho}{d\tau}\frac{dx^\sigma}{d\tau} = 0 \quad -(11)$$

and in the Newtonian limit this becomes:

$$\underline{a} = \underline{0} \quad -(12)$$

where $\underline{a}$ is the acceleration. Therefore in the Newtonian limit the geodesic equation is Newton's force law for $\underline{f} = m\underline{a} = \underline{0}$.

When there is a force present the right hand side of eq. (11) is no longer zero. The Lorentz force for example is (Carroll chapter 3):

$$f^\mu = e\, U^\lambda F_\lambda{}^\mu = e F^\mu{}_\nu \frac{dx^\nu}{d\tau} \quad -(13)$$

where $F^\mu{}_\nu$ is the electromagnetic field tensor and where U^λ is the four-velocity.

Thus, free particles in ECE field theory move along geodesics.

4) Development into ECE Theory

Develop eq. (2) into the covariant exterior derivative along a path:

$$\boxed{\frac{D\wedge}{d\lambda} := \frac{dx^\mu}{d\lambda} D\wedge} \quad -(14)$$

where:

$$D\wedge = d\wedge + \omega\wedge \quad -(15)$$

in the usual index suppressed notation. The first and second Cartan structure equations are:

$$T = D\wedge q = d\wedge q + \omega\wedge q \quad -(16)$$

$$R = D\wedge\omega = d\wedge\omega + \omega\wedge\omega. \quad -(17)$$

The ECE Ansatzen are:

$$A = A^{(0)} q \quad -(18)$$

$$F = A^{(0)} T. \quad -(19)$$

In eq. (16) T denotes the Cartan torsion, q the tetrad and ω the spin connection. The symbol $d\wedge$ is the Cartan exterior derivative. In eq. (17) R is the Cartan curvature form, in eq. (18) A is the electromagnetic potential form, $cA^{(0)}$ is the primordial voltage of

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ECE theory and F is the electromagnetic field form. The SI units of $A^{(0)}$ are $J\,s\,C^{-1}m^{-1}$ and the units of $cA^{(0)}$ are $\boxed{\text{volts} = J\,C^{-1}}$.

* **Therefore voltage is available from spacetime.** *

Restoring the indices for the electromagnetic potential form it is seen that parallel transport of A^a_μ is:

$$\frac{DA^a_\mu}{d\lambda} = 0 \quad -(20)$$

where:

$$A^a_\mu = A^{(0)} q^a_\mu . \quad -(21)$$

Eq. (20) means that:

$$\boxed{\frac{dx^\sigma}{d\lambda} D_\mu q^a_\nu = 0.} \quad -(22)$$

This is the equation for the parallel transport of the tetrad. However, from the tetrad postulate we know that:

$$D_\mu q^a_\nu = 0 \quad -(23)$$

so eq. (22) is true for any $dx^\sigma / d\lambda$.

The tetrad is constant along any curve $x^\mu(\lambda)$

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From eq. (21) it is concluded that the electromagnetic potential form A^a_μ is constant along any curve. Therefore the parallel transport of A^a_μ preserves the norm of vectors, sense of orthogonality, and so on. The condition:

$$D_\mu A^a_\nu = 0 \qquad -(24)$$

is the most general form of a "gauge condition"

Thus, for any $dx^\sigma / d\lambda$:

$$\boxed{\frac{DA^a_\mu}{d\lambda} = 0} \qquad -(25)$$

which is the geodesic equation of the electromagnetic potential. This equation defines the path followed by a free electromagnetic ~~field~~ potential.

The geodesic equation of ~~the~~ free electromagnetic field is:

$$\boxed{\frac{D \wedge A}{d\lambda} = \frac{dx^\mu}{d\lambda} D \wedge A = \frac{dx^\mu}{d\lambda} F = 0} \qquad -(26)$$

7)

where F obeys the first Bianchi identity:

$$d \wedge F = \mu_0 j = \frac{A^{(0)}}{\mu_0} \left(R \wedge q - \omega \wedge T \right) \quad -(27)$$

and its Hodge dual:

$$d \wedge \tilde{F} = \mu_0 J = \frac{A^{(0)}}{\mu_0} \left(\tilde{R} \wedge q - \omega \wedge \tilde{T} \right) \quad -(28).$$

Thus, for the free electromagnetic field:

$$\frac{dx^\mu}{d\lambda} F = 0. \quad -(29)$$

Restoring the indices on F:

$$\boxed{\frac{dx^\nu}{d\tau} F^{a\mu}{}_\nu = 0} \quad -(30)$$

where λ has been replaced by the proper time τ. Comparing (13) and (30) it is deduced that the Lorentz force is:

$$\boxed{f^{a\mu} = e \frac{dx^\nu}{d\tau} F^{a\mu}{}_\nu} \quad -(31)$$

For free particles (unaccelerated particles) eq. (30) shows that the Lorentz force is zero.

8) In eq (31):

$$F^{a\mu}{}_{\nu} = g^{\mu\rho} F^{a}{}_{\rho\nu} \quad -(32)$$

where $g^{\mu\rho}$ is the metric, and where:

$$F^{a}{}_{\rho\nu} = A^{(0)} T^{a}{}_{\rho\nu}. \quad -(33)$$

Here $T^{a}{}_{\rho\nu}$ is the Cartan torsion form written out with all indices.

$$\boxed{\text{Therefore the Lorentz force is the covariant exterior derivative of the potential form along any path in ECE space-time.}}$$

The metric used in EH theory is:

$$g_{\mu\nu} = q^{a}_{\mu} q^{b}_{\nu} \eta_{ab} \quad -(34)$$

where η_{ab} is the Minkowski metric of the tangent spacetime, so the Newtonian force can also be expressed in terms of the tetrad. This will be the subject of 56(2)