# GENERALLY COVARIANT DYNAMICS

by

Myron W. Evans,

Alpha Institute for Advanced Study (A.I.A.S)

(www.aias.us, www.atomicprecision.com, emyrone@aol.com)

ABSTRACT

Generally covariant translational and rotational dynamics are developed on the basis of Einstein Cartan Evans (ECE) field theory. Translational or central dynamics are defined as the limit of vanishing Cartan torsion where the Einstein Hilbert (EH) theory of gravitation is recovered. Rotational dynamics are defined in the limit where the translational Riemann curvature form vanishes and where the rotational Riemann form is dual to the Cartan torsion form. The mutual influence of translation and rotation is defined by the two Cartan structure equations and the two Bianchi identities of differential geometry. The equations of generally covariant rotational and translational dynamics are developed in the same form as the equations of generally covariant electrodynamics.

Keywords: Einstein Cartan Evans (ECE) field theory, generally covariant dynamics, generally covariant electrodynamics.

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## 1. INTRODUCTION

Classical dynamics has been developed continuously for more than four hundred years. Major advances occurred in the sixteenth and seventeenth centuries {1}, notably by Galileo, Brahe, Kepler and Newton, who synthesized the laws of classical translational dynamics. Later, rotational dynamics were developed by Euler and Coriolis, who inferred accelerations not present in Newtonian dynamics. Notable contributions came from Lagrange, Laplace and Hamilton using variational calculus. Following upon the results of the Michelson Morley experiment, length contraction was suggested by Fitzgerald, and developed into the theory of special relativity with notable contributions from Lorentz, Poincaré, Einstein and several others. Einstein inferred the theory of translational special relativity in 1905 and developed it into the theory of translational general relativity. The Einstein Hilbert (EH) field equation of translational dynamics was inferred independently by Einstein and Hilbert, and published in 1916. Later, Einstein and Cartan corresponded on the need for incorporating Cartan torsion into general relativity. In this paper the two Cartan structure equations and the two Bianchi identities of differential or Cartan geometry are inferred to be the equations of generally covariant dynamics, in which both translational and rotational motions are considered. These dynamics are valid in any frame of reference moving arbitrarily with respect to any other frame. The equations of these dynamics are therefore generally covariant as required by the theory of relativity and by objective natural philosophy.

In Section 2 the two Cartan structure equations and the two Bianchi identities are developed in a form which is identical to the equations of generally covariant electrodynamics {2-15} within a scalar factor $A^{(0)}$, essentially a primordial voltage. EH translational dynamics are defined as the limit where the Cartan torsion form vanishes and rotational dynamics are defined as the limit where the translational Riemann form vanishes. For rotational dynamics, the rotational Riemann form is dual to the Cartan torsion form, and the spin connection is dual to the tetrad. In Section 3 the Newtonian equations and principle of equivalence are inferred from the second Bianchi identity with zero Cartan torsion and the Euler equation is inferred from the first Cartan structure equation. In Section 4 the equations of generally covariant translational and rotational dynamics are developed in vector notation in the same form as the equations of generally covariant electrodynamics. Generally covariant dynamics provides several new inferences and suggests several phenomena not present in the EH limit. These may be tested with respect to cosmological anomalies where EH theory is not sufficient. There appear resonance solutions in both generally covariant dynamics and electrodynamics, and the ECE theory also gives the equations needed to describe the interaction of gravitation and electrodynamics.

## 2. THE EQUATIONS OF GENERALLY COVARIANT DYNAMICS

The equations of classical dynamics are given by ECE theory in any frame of reference moving arbitrarily with respect to any other frame. In a condensed notation with all indices suppressed for clarity {2-15} the equations of motion are given by Cartan geometry:

$$T = D \wedge q \quad -(1)$$

$$R = D \wedge \omega \quad -(2)$$

$$D \wedge T = R \wedge q \quad -(3)$$

$$D \wedge R = 0, \quad -(4)$$

$$D \wedge = d \wedge + \omega \wedge . \quad -(5)$$

Here T is the torsion form, q is the tetrad form, $\omega$ is the spin connection, R is the Riemann form and D denotes the covariant external derivative of Cartan {2-15}. This notation is fully explained and developed elsewhere {2-15} in form, tensor and vector notation. The condensed indexless notation of Eqs. (1) to (4) gives the basic structure most clearly. These equations of geometry are transformed into equations of classical dynamics using the Einstein Ansatz:

$$R = -kT \qquad -(6)$$

where in Eq. ( 6 ), R denotes scalar curvature, k is the Einstein constant and T is the index contracted canonical energy momentum tensor {16}. In general T contains contributions from all four fundamental fields (gravitational, electromagnetic, weak and strong). In the EH theory only the gravitational contribution is considered.

In the notation of Eqs ( 1 ) to ( 5 ) the EH field theory of 1916 is:

$$T = 0, \qquad -(7)$$

$$R \wedge q = 0, \qquad -(8)$$

$$D \wedge R = 0. \qquad -(9)$$

In this limit therefore the torsion form vanishes. Eq. ( 8 ) is the Ricci cyclic equation of EH theory. In tensor notation Eq. ( 8 ) is the familiar cyclic combination of Riemann tensors:

$$R_{\sigma\mu\nu\rho} + R_{\sigma\rho\mu\nu} + R_{\sigma\nu\rho\mu} = 0. \qquad -(10)$$

Eq. ( 9 ) is the second Bianchi identity. In tensor notation it becomes:

$$D^{\mu} G_{\mu\nu} = 0 \qquad -(11)$$

where $G_{\mu\nu}$ is the Einstein tensor:

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}. \qquad -(12)$$

In Eq. ( 12 ) $R_{\mu\nu}$ is the Ricci tensor and $g_{\mu\nu}$ is the symmetric metric tensor. The EH field equation is obtained from the second Bianchi identity ( 11 ) and the Noether Theorem:

$$D^{\mu}T_{\mu\nu} = 0. \quad -(13)$$

From Eqs. ( 11 ) and ( 13 ) we obtain the EH field equation:

$$G_{\mu\nu} = k T_{\mu\nu}. \quad -(14)$$

Here $T_{\mu\nu}$ is the canonical energy momentum tensor, whose index contracted form in EH theory is:

$$T = g^{\mu\nu} T_{\mu\nu}. \quad -(15)$$

It can be seen that the EH theory is limited by the omission of the Cartan torsion form and is therefore confined to the pure translational part of the general equations of dynamics, Eqs. ( 1 ) to ( 5 ). Pure rotational dynamics are defined as {2-15}:

$$T = D \wedge q, \quad -(16)$$

$$d \wedge T = 0, \quad -(17)$$

and it is seen that the translational part of the Riemann or curvature form is zero. Pure rotation is defined by:

$$R \wedge q = \omega \wedge T \quad -(18)$$

which means that the Cartan torsion is dual to the rotational part of the Riemann form. In this limit the Cartan torsion is the vector valued two-form dual to the rotational Riemann form, a tensor valued two-form. This duality is defined in the tangent (Minkowski) spacetime {2-15} of differential geometry and is analogous to the type of duality between for example an antisymmetric tensor in three dimensions and an axial vector in three dimensions. The rotational Riemann form is the dual of the Cartan torsion form. The translational Riemann form on the other hand is not dual to the Cartan torsion form. The translational Riemann form may therefore be zero when the Cartan form is non-zero and vice-versa. In EH theory only the translational Riemann form is considered, the rotational Riemann form and the Cartan torsion form are not considered in EH theory. In the latter the connection is the symmetric Christoffel connection and EH theory does not consider the interaction between rotation and translation. The equations needed to describe this interaction are Eqs. ( 1 ) to ( 5 ).

In the notation {15} of standard differential geometry Eqs. ( 1 ) to ( 4 ) become:

$$T^a = d \wedge q^a + \omega^a{}_b \wedge q^b, \quad -(19)$$

$$R^a{}_b = d \wedge \omega^a{}_b + \omega^a{}_c \wedge \omega^c{}_b, \quad -(20)$$

$$d \wedge T^a + \omega^a{}_b \wedge T^b = R^a{}_b \wedge q^b, \quad -(21)$$

$$d \wedge R^a{}_b + \omega^a{}_c \wedge R^c{}_b - R^a{}_c \wedge \omega^c{}_b = 0, \quad -(22)$$

in which the indices are those of the tangent (Minkowski) spacetime at point P to the base manifold. The indices of the base manifold are Greek indices which are always the same on both sides of any equation of Cartan geometry. So in the standard notation {15} of Cartan geometry the Greek indices are not written out. In refs. (2) to (14) however the equations of Cartan geometry are written out in full in form, tensor and vector notation.

The equations of Cartan geometry may be written as follows in the same overall format as the ECE equations of electrodynamics {2-14}. The first and second Cartan structure equations can be written as differential field equations in which the left hand side involves only the Cartan exterior derivative, and the right hand side is a combination of terms defining currents or source terms. Thus Eqs. ( 21 ) and ( 22 ) can be written as:

$$d \wedge T^a = j^a = R^a{}_b \wedge q^b - \omega^a{}_b \wedge T^b, \quad -(23)$$

$$d \wedge R^a{}_b = j^a{}_b = R^a{}_c \wedge \omega^c{}_b - \omega^a{}_c \wedge R^c{}_b. \quad -(24)$$

Here $j^a$ is the homogeneous current of ECE electrodynamics within the factor $A^{(0)}$.

Therefore dynamics and electrodynamics both originate in Cartan geometry, and are unified.

The Hodge duals {2-14} of Eqs. ( 23 ) and ( 24 ) are:

$$d \wedge \tilde{T}^a = J^a, \quad -(25)$$

$$d \wedge \tilde{R}^a{}_b = J^a{}_b, \quad -(26)$$

and $J^a$ is the inhomogeneous current of ECE electrodynamics within the factor $A^{(0)}$.

Similarly the two structure equations ( 19 ) and ( 20 ) may be written as follows:

$$d \wedge q^a = j_1^a = T^a - \omega^a{}_b \wedge q^b, \quad -(27)$$

$$d \wedge \omega^a{}_b = j_1{}^a{}_b = R^a{}_b - \omega^a{}_c \wedge \omega^c{}_b, \quad -(28)$$

whose Hodge duals are:

$$d \wedge \tilde{q}^a = J_1^a, \quad -(29)$$

$$d \wedge \tilde{\omega}^a{}_b = J_1{}^a{}_b. \quad -(30)$$

Eqs. ( 23 ), ( 24 ), ( 25 ) and ( 26 ) may be written in tensor and vector notation, producing much novel information on dynamics and cosmology. Later in this paper it will be shown that Newtonian dynamics is a limit of the vector equation:

$$\underline{\nabla} \cdot \underline{R}^a{}_b(\text{orbital}) = j^{a(0)}_b \quad -(31)$$

an equation which is one out of two vector equations given by the single form equation (26).

## 3. NEWTON AND EULER EQUATIONS

In Eq. (31), $R^a{}_b$ is the orbital part of the translational Riemann form, a quantity which is defined fully in Section 4. In this section the Newton inverse square law is derived straightforwardly from Eq. (31). First recognize that the Newtonian force $\underline{F}$ is derived from the force form $F^a{}_b$ defined by:

$$R^a{}_b\,(\text{orbital}) = \frac{kR}{c^2}\,F^a{}_b. \qquad -(32)$$

Here R is the scalar curvature of Cartan geometry {2-14} and c is the speed of light in vacuo, the universal constant of relativity theory. The force form is:

$$F^a{}_b = m_1\, g^a{}_b, \qquad -(33)$$

where $m_1$ is a scalar quantity with the units of kilograms. Thus $m_1$ is recognized as mass and $g^a{}_b$ is a tensor valued two-form with the units of acceleration. Eq. (31) is mathematically equivalent to:

$$\underline{F}^a{}_b = -\frac{G\, m_1 m_2}{r^2}\,\underline{k}^a{}_b = m_1\,\underline{g}^a{}_b \qquad -(34)$$

where G is the Newton gravitational constant defined by:

$$k = \frac{8\pi G}{c^2} \qquad -(35)$$

and where r is the distance between two masses $m_1$ and $m_2$. Restoring the indices of the base manifold to equation (33) gives:

$$F^a{}_{b\mu\nu} = m_1 g^a{}_{b\mu\nu}, \quad -(36)$$

an equation which shows that the force $F^a{}_{b\mu\nu}$ and the acceleration $g^a{}_{b\mu\nu}$ are both proportional to the orbital part of the translational Riemann form. In the Newtonian limit the base manifold approaches a Minkowski spacetime, so:

$$\underline{F}^a{}_b = \underline{F}, \quad \underline{g}^a{}_b = \underline{g}, \quad -(37)$$

and Eq. ( 34 ) becomes the Newton inverse square law:

$$\underline{F} = -\frac{G m_1 m_2}{r^2} \underline{k}, \quad -(38)$$

and Eq. ( 33 ) becomes the Newton force law:

$$\underline{F} = m_1 \underline{g}. \quad -(39)$$

Both laws are derived from the orbital part of the translational Riemann form of Cartan geometry, and this is the principle of equivalence of inertia and acceleration. The principle of equivalence states that must exist a scalar quantity $m_1$. It may be seen from Eq. ( 32 ) that mass does not enter into the relation between force and curvature, and this is the result of the well known Galileo experiment where two different masses dropped from the same height in the Earth's gravitational field hit the ground at the same time from the apocryphal Tower of Pisa. (In fact {1} Galileo proved this result using inclined planes.)

This simple derivation of Newtonian dynamics is confirmed as follows with reference to the well known Schwarzschild metric (SM), which by Birkhoff's Theorem is the unique spherically symmetric vacuum solution of the EH field equation ( 14 ). The SM is thus a solution of pure translational dynamics and takes no account of torsion in cosmology.